Computability Assignment
Year 2012/13 - Number 5

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1 Question

(I am re-proposing this exercise since only a few students solved it. This exercise is rather important, since it involves a reasoning which frequently appeared in past exam questions. While we shall see more examples of these concepts in class, it would be useful to start exercising on that. If you have already sumbitted an answer, skip this and do *not* resubmit your answer please.)

Let $\mathcal{F}$ be the set of partial functions $\{f \in (\mathbb{N} \rightsquigarrow \mathbb{N}) | \forall x \in \mathbb{N}.\ f(2 \cdot x) = x\}$.

- Define two distinct partial functions f_1, f_2 which belong to $\mathcal{F}$. (I.e, provide two such examples.)
- Define two distinct partial functions g_1, g_2 which do *not* belong to $\mathcal{F}$. (I.e, provide two such examples.)
- Define a partial function $f \in \mathcal{F}$, and consider the set of its *finite* restrictions $\mathcal{G} = \{g \in (\mathbb{N} \rightsquigarrow \mathbb{N}) | g \subseteq f \ \wedge \ \mathsf{dom}(g) \text{ finite}\}$.
  - Define two distinct partial functions h_1, h_2 which belong to $\mathcal{G}$. (I.e, provide two such examples.)
  - Prove whether $\mathcal{F} \cap \mathcal{G} = \emptyset$.

1.1 Answer

First part

- f_1 = { f(2.x) = x, x>=0

(RZ: I can't understand completely the notation, use $f(x) = \ldots$, not $f(2x) = \ldots$ or similar. Function f must be defined on all x, or if you want to make it partial and let it undefined on some points, state that explicitly so it's unambiguous.)

For all values of x greater than or equal to 0, the function f(2.x) would give us a value of x which belongs to the set of natural numbers.

For example if x=3, f(2.x) = 3, hence it holds

- f_2 = { f(2.x) = x, 1/x

For all values of 1/x, the function f(2.x) would give us a value of x which belongs to the set of natural numbers.

For example 1/x would lead to values of x cancelling hance ending with a value in the set of natural numbers.

Second part

- f_1 = { f(2.x) = x, x<0

For all values of x less than 0, the function f(2.x) would give us a value of x which is negative.

For example if x=-3, f(2.x) = -3, hence this is not in the set of natural numbers, hence the function f_1 does not belong to F.

- f_2 = { f(2.x) = x, 1/nx, where n belongs to the set of natural numbers.

For all values of 1/nx, the function f(2.x) would give us a value of 2/n which does not belongs to the set of natural numbers.

Third part

- h_1 = { square root of x, x=perfect squares of n, and n is in the set of natural numbers

If x is a perfect square of n, like 25, the result would be 5 which is a correct result.

- h_2 = { 9x, x=multiples of 9

If x is a multiple of 9, like 18, the result would be 18/9 which gives us 2, which is a correct result.

2 Question

Consider the following function:

$$f(n) = \sum_{i=0}^{n} i^2 + i$$

Write a FOR loop implementing f, then translate it in the λ-calculus as program F_1.

Then, write a recursive Java-like function implementing f, and translate it in the λ-calculus as program F_2.

2.1 Answer

sum:=0;
for i=0 to n do
sum:=i*i+1;
i:=i+1;
sum;
(RZ: this program looks very wrong to me: you want "sum:=sum+...", also the last line is not a command)

3 Question

Consider the following function:

$$f(n) = \begin{cases} x^2 + y & \text{if } n = \mathsf{pair}(\mathsf{inL}(x), y) \\ x + 4 \cdot y & \text{if } n = \mathsf{pair}(\mathsf{inR}(x), y) \end{cases}$$

Convince yourself that f is defined for all naturals n, i.e. it is total.

Write a λ-term implementing function f, exploiting the programs $Pair, Proj1, Proj2, InL, InR, Case, \ldots$ we saw in class (also defined in the notes).

3.1 Answer

Write your answer here.