

Some exercises forms (non exhaustive list)

- Given a judgment $\Gamma \vdash t : \tau$
with "holes", fill the holes

e.g. given Γ, τ , find t
" Γ, t , " τ

ex:

$x : ? \vdash x(?) : \forall \alpha. ?$

- Construct a typing derivation
for $\Gamma \vdash t : \tau$

(for the λ -calculi where we have
seen the rules)

- find the β / η / $\beta\eta$ normal form
of some term t

- prove that $\nexists t. \vdash t : \tau$

e.g. because τ leads to \perp (& consistency)

" τ leads to LEM

- theory questions:

provide definitions / statements / proofs
(those in the notes, only)

- prove a property on τ , provided $\tau: \forall \alpha. \tau(\alpha) \rightarrow \sigma(\alpha)$, and parametricity
- state the "free theorem" (in its naturality form) for some polymorphic τ
- prove $\tau \cong \sigma$ for some types
 - ↳ possibly, this can involve Yoneda
- prove that there is only one/two/-- values for a type τ , under certain assumptions about the model
- verify some basic category theoretic notion: eg.
 - check F is a functor
 - check η is natural
 - check \mathcal{C} is a category
 - ...
- prove some objects to be isomorphic $A \cong B$
- prove two morphisms equal
- Given $F: |\mathcal{C}| \rightarrow |\mathcal{D}|$ defined on objects only, extend the definition to a functor $\mathcal{C} \rightarrow \mathcal{D}$
 - Eg. $FX = X \times \mathbb{N} \times X$

- prove X initial

Note: some exercises may require

→ basic ← properties of groups / vector
spaces / topological spaces / sets

- state the induction principle
on some recursive type

- use "cata g ", at least
informally, on some type
e.g. use it to define some
program computing some given
function

Note: it is NOT required to know
the syntax of Haskell / Coq /
Java / Scala / ...

It is required only to have a good
intuition about the ideas
showcased in the examples we
saw

- give the type for the eliminator
of a (recursive) type

- ... and the type for the dependent
eliminator

- given a Coq dependent match,
describe intuitively why it
type checks

- encode τ in a type system
which allows (e.g.) only $\forall \alpha, \rightarrow$

Exg- $\tau = \mu \alpha. F \alpha$

$$\rightsquigarrow \tau \cong \forall \alpha. (F \alpha \rightarrow \alpha) \rightarrow \alpha$$

$$\cong \dots$$