The Equality Relation

A "dependent" predicate

$$\_ = \_ : \prod_{\alpha:*} \alpha \to \alpha \to *$$

Judgmental equality ($\equiv$) VS
Propositional equality ($=$)

($\equiv$) is "equality by definition, and $\to_\beta$ $\underbrace{(\text{or } \to_\eta)}_{\text{in some logics}}$ steps"

($\equiv$) is <u>easy</u> to check : to decide ($t \equiv e$), we reduce to normal form, and compare.

$$E_x : y:\mathbb{N} \vdash (\lambda x:\tau. x)\, y \equiv y$$

($=$) is the "real" equality : to establish ($t = e$) we need a <u>proof</u>, which is not trivial to write, in general

$$E_z : x:\mathbb{N}, y:\mathbb{N} \vdash \ ? : x + y = y + x$$

Equality can be formalized / axiomatized in many ways.

Typically:

- intro : $\mathrm{refl} : \prod_{\alpha:*} \prod_{x:\alpha} x = x$
- elim : "J" eliminator (complex!)

We see a few examples
in Coq (demo)