Functor Examples:

$F : Set \to Set$

$FX = X \times \mathbb{N}$

$h : A \to B \qquad Fh : FA \to FB$

$\qquad\qquad\qquad A \times \mathbb{N} \to B \times \mathbb{N}$

$\qquad\qquad\qquad (a, n) \mapsto (h(a), n)$

$F\,id = id \qquad F(h;g) = Fh \,;\, Fg$

$F : Set \to Set$

$FX = (\mathbb{N} \to X)$

$h : A \to B$

$Fh : (\mathbb{N} \to A) \to (\mathbb{N} \to B)$

$\qquad g \longmapsto g;h$ "post-composition"

$F : Set^{op} \to Set$ "contravariant functor"

$FX = (X \to \mathbb{N})$

$h \in Set^{op}(A, B) = Set(B, A)$

$Fh : FA \longrightarrow FB$

$\qquad (A \to \mathbb{N}) \to (B \to \mathbb{N})$

$\qquad g \longmapsto h;g$ "pre-composition"

$F : Set \to Set$

$FX = \{ \langle x_1, \dots, x_n \rangle \mid n \geq 0, \forall i. x_i \in X \}$

$Fh(\langle x_1, \dots \rangle) = \langle h(x_1), \dots \rangle$

"Forgetful" Functor

$F : \mathrm{Grp} \longrightarrow \mathrm{Set}$

$G \longmapsto |G|$

$h \longmapsto h$ as a function

Ditto for Top, $\mathrm{Vect}_{\mathbb{K}}$

Top° cat.

$|\mathrm{Top}^{\circ}| = (X, x)$ X top. space, $x \in X$

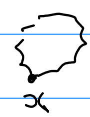
$\mathrm{Top}^{\circ}((X,x),(Y,y)) =$

$= \{ f : X \to Y \mid f \text{ cont} \wedge f(x) = y \}$

$F : \mathrm{Top}^{\circ} \longrightarrow \mathrm{Grp}$

$F(X, x) = \pi_1(X, x)$

fundamental group



$h : (X, x) \to (Y, y)$

$Fh : F(X,x) \to F(Y,y)$

$[g] \longrightarrow [h \circ g]$

$F : \mathrm{Set} \to \mathrm{Set}$

$FX = X \times X$

$Fh : FX \to FY$ if $h : X \to Y$

$(x_1, x_2) \mapsto (h(x_1), h(x_2))$

$\mathcal{C}$ cat s.t. $\mathcal{C}(A,B)$ is a set
"locally small"
(e.g. Set, Grp, Top, $\text{Vect}_{\mathbb{K}}$)
$C \in \mathcal{C}$
$F : \mathcal{C} \longrightarrow \text{Set}$ "representable functor"
$FX = \mathcal{C}(C, X)$
$h : A \longrightarrow_{\mathcal{C}} B \quad Fh : FA \longrightarrow_{\text{Set}} FB$
$\mathcal{C}(C,A) \to \mathcal{C}(C,B)$
$g \longmapsto g ; h$

$F : \mathcal{C}^{op} \longrightarrow \text{Set}$
$FX = \mathcal{C}(X, C)$
$h : A \longrightarrow_{\mathcal{C}^{op}} B \qquad h : B \longrightarrow_{\mathcal{C}} A$

$Fh : FA \longrightarrow FB$
$\mathcal{C}(A,C) \longrightarrow \mathcal{C}(B,C)$
$g \longmapsto h ; g$

$\mathcal{C}, \mathcal{D}$ cat $\Rightarrow$ $\mathcal{C} \times \mathcal{D}$ cat
$|\mathcal{C} \times \mathcal{D}| = |\mathcal{C}| \times |\mathcal{D}|$
$\mathcal{C} \times \mathcal{D}((C_1, D_1), (C_2, D_2)) =$
$\mathcal{C}(C_1, C_2) \times \mathcal{D}(D_1, D_2)$

Diagonal functor $F : \mathcal{C} \to \mathcal{C} \times \mathcal{C}$
$FC = (C, C)$
$h : A \to B \quad Fh : (A,A) \to (B,B)$
$Fh = (h, h)$

$I : \mathcal{C} \to \mathcal{C}$

$IX = X \qquad Ih = h$

$K_C : \mathcal{D} \to \mathcal{C} \qquad C \in |\mathcal{C}|$

$K_C(D) = C$

$K_C(h) = id_C$

$\mathcal{C}$ loc. small

$H : \mathcal{C}^{op} \times \mathcal{C} \to Set$ "Hom functor"

$H(A, B) = \mathcal{C}(A, B)$

$h : (A, B) \longrightarrow (A', B')$

$\sqsubseteq (h_1, h_2) \qquad h_1 : A' \longrightarrow A$

$\qquad\qquad\qquad h_2 : B \to B'$

$Fh : H(A, B) \to H(A', B')$

$\qquad g : A \to B \mapsto h_1 ; g ; h_2$

$(A, \sqsubseteq)$ preorder $\quad X \subseteq A$

$a \in A$ lower bound of X

$\equiv \forall x \in X.\ a \sqsubseteq x$

Cat th: lower bound $\rightsquigarrow$ cone

Def cone (simplified!)

$\mathcal{C}$ cat, $\mathcal{X} \subseteq |\mathcal{C}|$

a cone over $\mathcal{X}$ is a pair (C, f_{-})

$C \in |\mathcal{C}|$, $\forall X \in \mathcal{X}.\ f_X : C \to X$

$\mathrm{Cone}_{\mathcal{X}}$ is a cat !