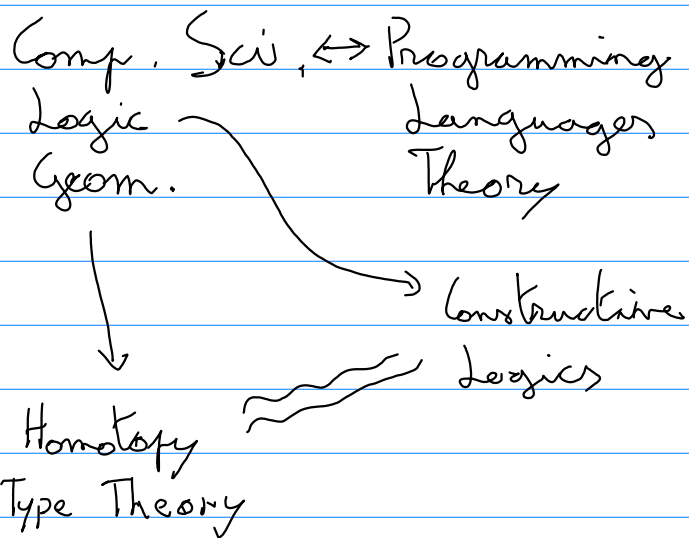


Type Theory



Set Th vs Type Th.

Frege: naive set th

- 1) p predicate
 $\hookrightarrow \exists A = \{x \mid p(x)\}$
- 2) $y \in \{x \mid p(x)\} \Leftrightarrow p(y)$
(extensionality)

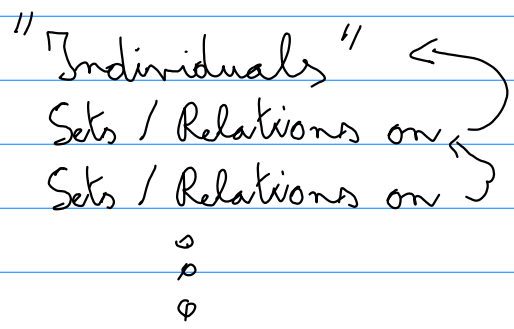
Russell's Paradox

$$\begin{cases} A = \{x \mid x \notin x\} \\ \text{(ext)} \\ \forall y. y \in A \Leftrightarrow y \notin y \\ \Rightarrow A \in A \Leftrightarrow A \notin A \\ \hookrightarrow \text{contradiction!} \end{cases}$$

Russell's Type Theory

(informal ~)

↳ Constrain valid predicates



(Relations had ramified types)

"If X is defined in terms of all the elements of Y then X can not be a part of Y"
 vicious-circle principle



" $x \notin x$ " is meaningless

$$A = \{ m \in \mathbb{N} \mid \forall B \in \mathcal{P}(\mathbb{N}) \dots \}$$

