Computability Assignment
Year 2013/14 - Number 7

Please keep this file anonymous: do not write your name inside this file.

More information about assignments at http://disi.unitn.it/∼zunino/teaching/computability/assignments

Please do not submit a file containing only the answers; edit this file, instead, filling the answer sections.

1 Question

A set $C \subseteq \mathbb{N}$ is called *upward closed* iff $\forall x \in C.\ \forall y \in \mathbb{N}\ (y > x \Longrightarrow y \in C)$.

Provide a characterization of the set $Z = \{X | X \in \mathcal{P}(\mathbb{N}) \wedge X \text{ is upward closed}\}$ (i.e. find a property p such that $Z = \{X | X \in \mathcal{P}(\mathbb{N}) \wedge p(X)\}$, where p could be a conjunction of many "simpler" properties).

1.1 Answer

$p(x) = \forall y(y \notin X\ \vee\ (y+1) \in X)$

2 Question

A set $X \subseteq \mathbb{N}$ is called *cofinite* iff $\overline{\{X\}}$ is finite.

Prove or refute the statement: "if $X,\ Y \in \mathcal{P}(\mathbb{N})$ are NOT cofinite, then $X \cup Y$ is NOT cofinite".

2.1 Answer

If X respectively Y is not cofinite then $\overline{\{X\}}$ respectively $\overline{\{Y\}}$ is infinite. but we do not know about finiteness of $\overline{\{X \cup Y\}} = \overline{\{X\}} \cap \overline{\{Y\}}$ so statement is not valid. For istance, take $X = \{a | a\, is\, even\}$, $Y = \{a | a\, is\, odd\}$. $\overline{\{X\}}$ and $\overline{\{Y\}}$ are not finite, so X and Y are not cofinite. But $X \cup Y = \mathbb{N}$, so $\overline{X \cup Y} = \emptyset$ is finite so $X \cup Y$ is cofinite.

3 Question

In what follows, $A \subseteq \mathbb{N}$.

1. Prove that if there exists a bijection $f \in (\mathbb{N} \to A)$, then A is infinite.

2. Can you provide an example of an infinite set A and of a function $f \in (\mathbb{N} \to A)$ which is neither injective nor surjective?

3.1 Answer

1. For contraddiction assume that A is finite, and that there exist a bijection $f \in (\mathbb{N} \to A)$. Now suppose that we take B s.t. $|B| = |A| + 1$, $B \subset \mathbb{N}$. So $|f(B)| \leq |A|$, then there must be two elements $b_1, b_2 \in B$ s.t. $|f(b_1) = f(b_2)$. But then f is not injective. Contraddiction because our hypothesis was that f was bijection so A is infinite;

2. Take $A = \mathbb{N}$, $f(n) = 0$.