Computability Assignment
Year 2012/13 - Number 4

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1 Question

Let A, B be sets and suppose that $A \leftrightarrow B$ (i.e. there exists a bijection $f \in (A \to B)$). Show that for all sets C, $(C \to (A \times A)) \leftrightarrow (C \to (A \times B))$.

1.1 Answer

Since we have a bijection between $A$and B, we can transfrom the element of B in $(C \to (A \times A)) \leftrightarrow (C \to (A \times B))$ into an element of A, then the function we are looking for would just be the identity function.

2 Question

1. Doeas a surjective function $f \in (\mathbb{N} \to (\mathbb{N} \to \{0, 1, 2, 3\}))$ exist?
2. Does an injective function $f \in (\mathcal{P}(\mathbb{N}) \rightsquigarrow \mathbb{N})$ exist?
3. Does an injective function $f \in (\mathcal{P}(\mathbb{N}) \to \mathbb{N})$ exist?

Justify your answers.

2.1 Answer

1. No, suppose that f existed. Consider $g(x) = (f(x)(x) + 1)\%4$. Since f is surjective, and g a function in its range, it means that for some i, f(i)=g. But then f(i)(i)=g(i)=(f(i)(i)+1)%4, contradiction.
2. Yes, take $f(\{a\}) = a, a \in N$

3. No. Suppose that there was such a function. Then there would be a partial function $f^{-1} : N \rightsquigarrow P(N)$, that would have to be surjective (because f is a total function, therefore connects every element of P(N) to an element of N) as well as injective (becuse by hypothesis, f is injective, thus its inverse is an injective partial function). This means that the function $f_2^{-1} : A \to P(N)$ where A=range(f) is a bijection. But we already know that $P(A) \subseteq P(N)$, and from Cantor's argument we know that there does not exist a bijection between A and P(A), because the function is not surjective, but then $f_2^{-1} : A \to P(N)$ cannot be surjective either (because the codomain P(A) is a subset of P(N)), thus the function is not a bijection, contradiction.

3 Question

Let A, B be nonempty sets and let $f \in (A \to B)$. Define a function $g \in (B \rightsquigarrow A)$ such that $\mathrm{dom}(g) \neq \emptyset$ and for all $b \in$ dom(g), $(f \circ g)(b) = b$.

3.1 Answer

One needs only to take $g : B \to A$, where $dom(g) = range(f)$, defined as $g(b) = min(\{a | a \in A \wedge f(a) = b\})$. Note that C is always different from the empty set, because both A and B are non empty, and f is total (thus sends each element of A into an element of B).