Computability Assignment
Year 2012/13 - Number 4

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1 Question

Let A, B be sets and suppose that $A \leftrightarrow B$ (i.e. there exists a bijection $f \in (A \to B)$). Show that for all sets C, $(C \to (A \times A)) \leftrightarrow (C \to (A \times B))$.

1.1 Answer

By the formula $(C \to (A \times A)) \leftrightarrow (C \to (A \times B))$ can we see that we have a bijctive between $(C \to (A \times A))$ and $(C \to (A \times B))$ and so to verify that the given formula is true for all C we find a bijective formula $g \in (A \to B)$ such that we can replace the second element of pair $< a_1, a_2 > \in (A \times A)$ with $b \in B$ to obtain $< a_3, b > \in (A \times B)$ and vice versa i.e $g^{-1}(b) = a_2$. The function g has necessarily f since there is a bijection between A and B. As regards the bijection between A and the same A is always verified and so we can affirm that $\forall C.(C \to (A \times A)) \leftrightarrow (C \to (A \times B))$.

2 Question

1. Doeas a surjective function $f \in (\mathbb{N} \to (\mathbb{N} \to \{0, 1, 2, 3\}))$ exist?
2. Does an injective function $f \in (\mathcal{P}(\mathbb{N}) \rightsquigarrow \mathbb{N})$ exist?
3. Does an injective function $f \in (\mathcal{P}(\mathbb{N}) \to \mathbb{N})$ exist?

Justify your answers.

2.1 Answer

1. Yes because we can rewrite $f \in (\mathbb{N} \to (\mathbb{N} \to \{0,1,2,3\}))$ as $f \in f(g(x))$ where $g(x) = (\mathbb{N} \to \{0,1,2,3\})$ and $f(x) = \mathbb{N} \to (g(x))$. $g(x)$ is surjective since all elements of codomain are figures of domain and since the domain of $g(x)$ is equal of domain of $f(x)$ we can say that $f(x)$ is surjective.

2. Yes because since f is a partial function it is not necessary that all elements of the domain, in this case($\mathcal{P}(\mathbb{N})$, have a corresponding element in the codomain which in this case is $\mathbb{N}$.

3. No because f is a total function and then to be injective each element x must have a different $f(x)$ and this is not possible since the set ($\mathcal{P}(\mathbb{N})$ is by definition larger than $\mathbb{N}$.

3 Question

Let A, B be nonempty sets and let $f \in (A \to B)$. Define a function $g \in (B \rightsquigarrow A)$ such that $\text{dom}(g) \neq \emptyset$ and for all $b \in \text{dom(g)}$, $(f \circ g)(b) = b$.

3.1 Answer

Since g is a parzial function the domain of g shall not contain all element of B and so $dom(g) \subset B$. Assumed that $g = f^{-1}$ we can note that $ran(f) = dom(g) \subset B$. Than $\forall b \in dom(g).f(g(b)) = f(f^{-1}(b)) = b$ while $\forall b' \notin \text{dom}(g).f(f^{-1}(b'))$ is not defined.