Computability Assignment
Year 2012/13 - Number 4

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1 Question

Let A, B be sets and suppose that $A \leftrightarrow B$ (i.e. there exists a bijection $f \in (A \to B)$). Show that for all sets C, $(C \to (A \times A)) \leftrightarrow (C \to (A \times B))$.

1.1 Answer

If exists a function $(C \to (A \times A))$ it is possible to map the domain of this function to $(A \times B)$, because $A \leftrightarrow B$, and obtain $(C \to (A \times B))$.
The mapping is possible because $\forall a, y \in A.\, (a, y) \in A \times A. \exists! b = f(y). b \in B. (a, b) \in A \times B$
With the same reasoning is possible to prove the mapping from $B \times A$ to $A \times A$

2 Question

1. Doeas a surjective function $f \in (\mathbb{N} \to (\mathbb{N} \to \{0, 1, 2, 3\}))$ exist?

2. Does an injective function $f \in (\mathcal{P}(\mathbb{N}) \rightsquigarrow \mathbb{N})$ exist?

3. Does an injective function $f \in (\mathcal{P}(\mathbb{N}) \to \mathbb{N})$ exist?

Justify your answers.

2.1 Answer

1. Yes, it exists: $f(f(x)) = x - \lfloor \frac{x}{4} \rfloor 4$. Proof.
   $\forall y \in \{0,1,2,3\}\, \exists x \in \mathbb{N}$, namely $y + \lfloor \frac{x}{4} \rfloor 4$ such that
   $f(x) = f\left(y + \lfloor \frac{x}{4} \rfloor 4\right) = y + \lfloor \frac{x}{4} \rfloor 4 - \lfloor \frac{y + \lfloor \frac{x}{4} \rfloor 4}{4} \rfloor 4 = y + \lfloor \frac{x}{4} \rfloor 4 - \lfloor \frac{x}{4} \rfloor 4 = y$
   This proves that the function is surjective

2. Yes, it exists: f(x)$\begin{cases} x & if\ \{x\} \\ undefined & otherwise \end{cases}$

3. No, it doesn't exist. It can be proved using the diagonalization argument.

3 Question

Let A, B be nonempty sets and let $f \in (A \to B)$. Define a function $g \in (B \rightsquigarrow A)$ such that dom$(g) \neq \emptyset$ and for all $b \in$ dom(g), $(f \circ g)(b) = b$.

3.1 Answer

Let $A = \{1,2,3,4,5\}$ and $B = \{0,1,2,3\}$
$f(x) = \lfloor \frac{x}{4} \rfloor$
$g(y) = \begin{cases} y & y \geq 0 \\ undefined & otherwise \end{cases}$