Computability Assignment
Year 2012/13 - Number 3

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1 Question

Recall the notions of image and preimage of a set with respect to a function: formally, if $A \subseteq X$, then $f(A) = \{f(x)|x \in A\} \subseteq Y$ and that, if $B \subseteq Y$, then $f^{-1}(B) = \{x|x \in X \land f(x) \in B\} \subseteq X$. (Note that here A and B are not points in the domains of f, f^{-1}, but rather sets of such points)

1. For $A \subseteq X$, determine the relation $(\subseteq, =, \supseteq)$ between A and $f^{-1}(f(A))$.
2. For $B \subseteq Y$, determine the relation $(\subseteq, =, \supseteq)$ between B and $f(f^{-1}(B))$.
3. If $C \subset A \subseteq X$, is it always true that $f(C) \subset f(A)$?
4. If $C \subset B \subseteq Y$ and $f^{-1}(B) \neq \emptyset$, is it always true that $f^{-1}(C) \subset f^{-1}(B)$?

1.1 Answer

Write your answer here.

2 Question

Let A, B be sets, and let $\mathsf{id}_A, \mathsf{id}_B$ denote the identity functions over A and B respectively. Assume $f \in (A \to B)$ and $g \in (B \to A)$ be functions satisfying $g \circ f = \mathsf{id}_A$ and $f \circ g = \mathsf{id}_B$, where as usual $\circ$ denotes function composition. Prove that f is a bijection (i.e., injective and surjective).

2.1 Answer

Write your answer here.

3 Question

(This question is more challenging.) Find two functions $f, g \in (\mathbb{N} \to \mathbb{N})$ that satisfy all the following conditions:

1. $\mathsf{ran}(f) \neq \mathbb{N}$ and $\mathsf{ran}(g) \neq \mathbb{N}$;
2. $\mathsf{ran}(f)$ and $\mathsf{ran}(g)$ are infinite sets;
3. $\mathsf{ran}(h) = \mathbb{N}$ where $h(n) = f(n) + g(n)$;
4. $\exists n \in \mathbb{N}.\ \mathsf{ran}(g \circ f) = \{n\}$.

3.1 Answer

I claim that if we choose the two functions f as

$$f(n) = \begin{cases} n & if\, n\, is\, even \\ 0 & o.w. \end{cases}$$

and g as

$$g(n) = \begin{cases} n & if\, n\, is\, odd \\ 0 & o.w. \end{cases}$$

Points 1 and 2 can be easily check.
Point 3 is satisfied because we have that if n is even then the we get $h(n) = n + 0 = n$, while if it is odd we get $h(n) = 0 + n = n$. Therefore $ran(h) = \mathbb{N}$.
Point 4 is satisfied because $f(n)$ is returning either an even number or 0. On both these inputs the function g returns 0 and therefore the condition is satisfied.