

# Computability Assignment

## Year 2013/14 - Number 1

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### 1 Question

Define a binary property  $p(x, y)$  over natural numbers that satisfies both the requisites:

1.  $\forall x \in \mathbb{N}. \exists y \in \mathbb{N}. p(x, y)$  and
2. *it is false that*  $\forall y \in \mathbb{N}. \exists x \in \mathbb{N}. p(x, y)$

Provide a definition for  $p$ , and a proof for the above claims.

#### 1.1 Answer

One property can be  $p(x, y) = x < y$ .

$\forall x \in \mathbb{N}. \exists y \in \mathbb{N}. p(x, y)$  is true because always exists an  $y \in \mathbb{N}$  such that  $x < y$  because natural numbers  $\mathbb{N}$  are infinite.

But if we take  $y = 0$  we cannot find an  $x \in \mathbb{N}$  such that  $x < y$  so the requisite  $\forall y \in \mathbb{N}. \exists x \in \mathbb{N}. p(x, y)$  is FALSE.