Computability Assignment Year 2013/14 - Number 1

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1 Question

Define a binary property p(x, y) over natural numbers that satisfies both the requisites:

- 1. $\forall x \in \mathbb{N} : \exists y \in \mathbb{N} : p(x, y)$ and
- 2. it is false that $\forall y \in \mathbb{N} . \exists x \in \mathbb{N} . p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

One property can be p(x, y) = x < y.

 $\forall x \in \mathbb{N} : \exists y \in \mathbb{N} : p(x, y) \text{ is true because alwas exists an } y \in N \text{ such that } x < y \text{ because natural numbers } N \text{ are infinite.}$

But if we take y = 0 we cannot find an $x \in N$ such that x < y so the requisite $\forall y \in N. \exists x \in N. p(x, y)$ is FALSE.