

Computability Assignment

Year 2013/14 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers that satisfies both the requisites:

1. $\forall x \in \mathbb{N}. \exists y \in \mathbb{N}. p(x, y)$ and
2. *it is false that* $\forall y \in \mathbb{N}. \exists x \in \mathbb{N}. p(x, y)$

Provide a definition for p , and a proof for the above claims.

1.1 Answer

We can use De Morgan's laws:

$$(\neg \forall y. p(y)) \iff (\exists y. \neg p(y)) \text{ and}$$

$$(\neg \exists x. p(x)) \iff (\forall x. \neg p(x)) \text{ for the second claim.}$$

$$\neg(\forall y \in \mathbb{N}. \exists x \in \mathbb{N}. p(x, y)) \iff \exists y \in \mathbb{N}. \forall x \in \mathbb{N}. \neg p(x, y)$$

It is impossible to find p that satisfies $p(x, y) \wedge \neg p(x, y)$.