Computability Assignment
Year 2013/14 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers that satisfies both the requisites:

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$ and

2. *it is false that* $\forall y \in \mathbb{N}.\exists x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

We can use De Morgan's laws:

$(\neg\forall y.p(y)) \iff (\exists y.\neg p(y))$ and

$(\neg\exists x.p(x)) \iff (\forall x.\neg p(x))$ for the second claim.

$\neg(\forall y \in \mathbb{N}.\exists x \in \mathbb{N}.p(x, y)) \iff \exists y \in \mathbb{N}.\forall x \in \mathbb{N}.\neg p(x, y)$

It is impossible to find p that satisfies $p(x, y) \wedge \neg p(x, y)$.