Computability Assignment
Year 2013/14 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers that satisfies both the requisites:

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$ and

2. *it is false that* $\forall y \in \mathbb{N}.\exists x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

Given $x \in \mathbb{N}$ and $y \in \mathbb{N}$, $p(x, y)$ can be defined as $y = x^2$.

1. The first requisite is proved as $\forall x \in \mathbb{N}$ we can find $y = x^2 \in \mathbb{N}$.

2. The second requisite can be proved by showing an example.
   Take $y = 10 \in \mathbb{N}$ then the property $p(x, y)$ is not true as there is no natural number x such that $x^2 = 10$.