Computability Assignment
Year 2013/14 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers that satisfies both the requisites:

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$ and
2. *it is false that* $\forall y \in \mathbb{N}.\exists x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

definition $p(x, y) := (y = x)^3$

Proof

1. For any x, it's cube will always be a Natural Number ($x^3 = x * x * x$ and $x * x = \sum_{k=1}^{x} x$, so starting from a Natural Number it will be always a sum of Natural Numbers).
2. We need a counter-example, $y = 3 \in \mathbb{N}$, but $x = \sqrt[3]{3} \notin \mathbb{N}$