Computability Assignment
Year 2013/14 - Number 1

Please keep this file anonymous: do not write your name inside this file.

More information about assignments at http://disi.unitn.it/∼zunino/teaching/computability/assignments

Please do not submit a file containing only the answers; edit this file, instead, filling the answer sections.

1 Question

Define a binary property $p(x, y)$ over natural numbers that satisfies both the requisites:

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$ and
2. *it is false that* $\forall y \in \mathbb{N}.\exists x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

Let's define $p(x, y) = x < y$, in that case the both the requisites are satisfied:

1. For $x = 0$ the formula in 1 is satisfied because $\exists y \in \mathbb{N}.p(0, y)$ holds ($p(0, y)$ is satisfied $\forall y \in \mathbb{N} \setminus \{0\}$. For the general case indeed taking $x = n$ for a gemeric $n \in \mathbb{N}$ the property holds for $y = n + 1$ so the property 1 is satisfied.
2. For prooving that 2 holds we need to prove that $\forall y \in \mathbb{N}.\exists x \in \mathbb{N}.p(x, y)$ is false. In order to do that is enought to find an $y \in \mathbb{N}$ such that $\neg\exists x \in \mathbb{N}.p(x, y)$. For $y = 0$ in fact doesn't exist an $x \in \mathbb{N}$ such that $x < 0$ because $\mathbb{N} = \{0, 1, 2, ...\}$. So $\forall y \in \mathbb{N}.\exists x \in \mathbb{N}.p(x, y)$ is false and the the second property holds.