Computability Assignment
Year 2013/14 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers that satisfies both the requisites:

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$ and
2. *it is false that* $\forall y \in \mathbb{N}.\exists x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

A property satisfying the above conditions is$p(x, y) := (y = 2x)$. (1) is satisfied because if x belongs to $\mathbb{N}$, we can always compute 2x, which will be a natural number since $\mathbb{N}$ is closed under multiplication. To satisfy (2) we just need to supply a counterexample to the proposition $\forall y \in \mathbb{N}.\exists x \in \mathbb{N}.p(x, y)$, and as we can see it does not work with y $= 3$, or any odd number, for that matter (since x should be a natural number, and not, for example, a rational one, and y should be written as the product of x and 2, thus y%2=0).