Computability Assignment
Year 2013/14 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers that satisfies both the requisites:

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$ and

2. *it is false that* $\forall y \in \mathbb{N}.\exists x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

$p(x, y) := (\frac{x}{y} \in \mathbb{N})$

1. I split the proof in two cases:

   (a) If $x = 0$ then $(\frac{x}{y} \in \mathbb{N})$ is true $\forall y \in \mathbb{N}$ and $y \neq 0$, so in particolar y exists;

   (b) If $x \neq 0$ then I take $y = x$ so that $(\frac{x}{y} = \frac{x}{x} = 1 \in \mathbb{N})$.

2. It is obvious by taking $y = 0$.