Computability Assignment
Year 2013/14 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers that satisfies both the requisites:

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$ and
2. *it is false that* $\forall y \in \mathbb{N}.\exists x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

If we take $p(x, y) = x < y$ we have

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.x < y$
   proof:
   Suppose by contradiction that 1. is false, then
   $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.x < y \Leftrightarrow \neg(\exists x \in \mathbb{N}.\forall y \in \mathbb{N}.x \geq y)$
   but this means that x is the maximum number in $\mathbb{N}$ and $|\mathbb{N}| = x + 1$which is a contradiction. Therefore 1. must be true
2. $\forall y \in \mathbb{N}.\exists x \in \mathbb{N}.x < y$
   we just need to find a counter-example
   If we take $y = 0$ then $\nexists x \in \mathbb{N}.x < y$