

Computability Assignment

Year 2013/14 - Number 8

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1 Preliminaries

Recall that an equivalence relation \sim over a set A is a binary relation that satisfies all of the following:

1. $\forall x \in A. x \sim x$ (reflexivity);
2. $\forall x, y \in A. x \sim y \Rightarrow y \sim x$ (symmetry);
3. $\forall x, y, z \in A. x \sim y \wedge y \sim z \Rightarrow x \sim z$ (transitivity).

If A is a set and \sim is an equivalence relation over A , then for all $x \in A$ one can define the *equivalence class* of x with respect to \sim , that is the set $[x] = \{y \mid y \in A \wedge x \sim y\}$. We will denote by A/\sim the set of all equivalence classes of elements of A , that is $A/\sim = \{[x] \mid x \in A\}$.

2 Question

Let A be a set and \sim an equivalence relation over A . Show that, for all $x, y \in A$, either $[x] = [y]$ or $[x] \cap [y] = \emptyset$. Hint: remember that, by the *law of excluded middle*, for any choice of $x, y \in A$, either $x \sim y$ or $x \not\sim y$ (where $x \not\sim y$ means $\neg(x \sim y)$).

2.1 Answer

Write your answer here.

3 Question

Let $f \in (\mathbb{N} \rightarrow \mathbb{N})$. For each of the relations below, prove whether it is an equivalence relation over \mathbb{N} :

1. $x \sim y$ if and only if $f(x) = f(y)$;
2. $x \sim y$ if and only if $f(x) \neq f(y)$;
3. $x \sim y$ if and only if $f^{-1}(x) \cap f^{-1}(y) \neq \emptyset$.

3.1 Answer

Write your answer here.

4 Question

Let $\{\varphi_n\}_{n \in \mathbb{N}}$ be an enumeration for the set of recursive partial functions from \mathbb{N} to \mathbb{N} , and let \sim be the equivalence relation over \mathbb{N} defined as follows: $i \sim j$ if and only if $\varphi_i = \varphi_j$. Moreover, let $e \in (\mathbb{N} \times \mathbb{N} \rightsquigarrow \mathbb{N})$ the partial function defined as $e(a, b) = \varphi_a(b)$.

Prove that, if $i \sim j$, then $\forall b \in \mathbb{N}, e(i, b) = e(j, b)$.

4.1 Answer

Write your answer here.

5 Remark

Notice that, by what you have proved in the previous exercise, it can be deduced that one can obtain a well-defined partial function $f \in (\mathbb{N}/\sim \times \mathbb{N} \rightsquigarrow \mathbb{N})$ by posing $f([a], b) = e(a, b)$.