

Computability Assignment

Year 2013/14 - Number 6

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1 Question

Remember that for all $A \subseteq \mathbb{N}$, $\overline{A} = \mathbb{N} \setminus A$, and id_A is the identity function on A .

Let $f \in (\mathbb{N} \rightarrow \mathbb{N})$ and let $A = \{f(n) \mid n \text{ is a prime number}\}$.

1. Characterize the elements of the set \overline{A} (i.e. find a property p such that $\overline{A} = \{n \mid p(n)\}$). Notice that p could be a conjunction of many “simpler” properties.
2. Define a function $g \in (A \rightarrow \mathbb{N})$ such that $f \circ g = \text{id}_A$.

1.1 Answer

Write your answer here.

2 Question

Let $A = \{n \mid \exists m \in \mathbb{N}. n = m^2\}$ and $B = \{2n \mid n \in \mathbb{N}\}$. Following the steps outlined below, define a bijection $f \in (\mathbb{N} \rightarrow \mathbb{N})$ such that $f(A) = B$ and $f(\overline{A}) = \overline{B}$.

1. Provide a bijection $g \in (A \rightarrow \mathbb{N})$.
2. Provide a bijection $h \in (\mathbb{N} \rightarrow B)$.
3. Argue that there exists a bijection $g' \in (\overline{A} \rightarrow \mathbb{N})$.
4. Provide a bijection $h' \in (\mathbb{N} \rightarrow \overline{B})$.

5. Prove that the function $f \in (\mathbb{N} \rightarrow \mathbb{N})$ defined as

$$f(n) = \begin{cases} (h \circ g)(n) & \text{if } n \in A \\ (h' \circ g')(n) & \text{if } n \in \overline{A} \end{cases}$$

satisfies all the desired properties.

2.1 Answer

Write your answer here.