

# Computability Assignment

## Year 2012/13 - Number 3

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### 1 Question

Recall the notions of image and preimage of a set with respect to a function: formally, if  $A \subseteq X$ , then  $f(A) = \{f(x) | x \in A\} \subseteq Y$  and that, if  $B \subseteq Y$ , then  $f^{-1}(B) = \{x | x \in X \wedge f(x) \in B\} \subseteq X$ . (Note that here  $A$  and  $B$  are not points in the domains of  $f, f^{-1}$ , but rather sets of such points)

1. For  $A \subseteq X$ , determine the relation ( $\subseteq, =, \supseteq$ ) between  $A$  and  $f^{-1}(f(A))$ .
2. For  $B \subseteq Y$ , determine the relation ( $\subseteq, =, \supseteq$ ) between  $B$  and  $f(f^{-1}(B))$ .
3. If  $C \subset A \subseteq X$ , is it always true that  $f(C) \subset f(A)$ ?
4. If  $C \subset B \subseteq Y$  and  $f^{-1}(B) \neq \emptyset$ , is it always true that  $f^{-1}(C) \subset f^{-1}(B)$ ?

#### 1.1 Answer

Write your answer here.

### 2 Question

Let  $A, B$  be sets, and let  $\text{id}_A, \text{id}_B$  denote the identity functions over  $A$  and  $B$  respectively. Assume  $f \in (A \rightarrow B)$  and  $g \in (B \rightarrow A)$  be functions satisfying  $g \circ f = \text{id}_A$  and  $f \circ g = \text{id}_B$ , where as usual  $\circ$  denotes function composition. Prove that  $f$  is a bijection (i.e., injective and surjective).

#### 2.1 Answer

Write your answer here.

### 3 Question

(This question is more challenging.) Find two functions  $f, g \in (\mathbb{N} \rightarrow \mathbb{N})$  that satisfy all the following conditions:

1.  $\text{ran}(f) \neq \mathbb{N}$  and  $\text{ran}(g) \neq \mathbb{N}$ ;
2.  $\text{ran}(f)$  and  $\text{ran}(g)$  are infinite sets;
3.  $\text{ran}(h) = \mathbb{N}$  where  $h(n) = f(n) + g(n)$ ;
4.  $\exists n \in \mathbb{N}. \text{ran}(g \circ f) = \{n\}$ .

#### 3.1 Answer

Write your answer here.