

Computability Assignment

Year 2012/13 - Number 2

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1 Question

In this exercise, $p(x)$ and $q(x)$ will be two unary properties over natural numbers, and P and Q will denote the sets $P = \{x \in \mathbb{N} : p(x) \text{ holds}\}$ and $Q = \{x \in \mathbb{N} : q(x) \text{ holds}\}$. If possible, for each of the cases below find two properties $p(x)$ and $q(x)$ such that $\forall x \in \mathbb{N}. p(x) \Rightarrow q(x)$ and

1. $P \subset Q$ (strict inclusion);
2. $Q \subset P$ (strict inclusion);
3. $P \setminus Q \neq \emptyset$;
4. $Q \setminus P \neq \emptyset$.

If for some of the above cases it's impossible to find such properties, provide a brief explanation of why is it so.

1.1 Answer

Write your answer here.

2 Preliminaries

Given an infinite sequence of sets $(A_i)_{i \in \mathbb{N}}$, we define $\bigcap_{i=0}^{\infty} A_i = \bigcap \{A_i \mid i \in \mathbb{N}\} = \{x \mid \forall i \in \mathbb{N} x \in A_i\}$ and $\bigcap_{i=0}^k A_i = \bigcap \{A_i \mid i \in \mathbb{N} \wedge i \leq k\} = A_0 \cap A_1 \cap \dots \cap A_k$.

3 Question

Assume $(A_i)_{i \in \mathbb{N}}$ to be an infinite sequence of sets of natural numbers, satisfying

$$\mathbb{N} \supseteq A_0 \supseteq A_1 \supseteq A_2 \supseteq A_3 \cdots (*)$$

For each property p_i shown below, state whether

- the hypothesis $(*)$ is sufficient to conclude that p_i holds; or
- the hypothesis $(*)$ is sufficient to conclude that p_i does not hold; or
- the hypothesis $(*)$ is not sufficient to conclude anything about the truth of p_i .

Justify your answers (briefly).

1. p_1 : $\forall k \in \mathbb{N}. A_k = \bigcap_{i=0}^k A_i$;
2. p_2 : if $\forall i \in \mathbb{N}. A_i$ is finite, then there exists $j \in \mathbb{N}$ such that $A_j = A_{j+1}$;
3. p_3 : for all i , if A_i is finite, then $A_i = A_{i+1}$;
4. p_4 : if $\forall i \in \mathbb{N}. A_i \neq A_{i+1}$, then $\bigcap_{i=0}^{\infty} A_i = \emptyset$;
5. p_5 : if $\forall i \in \mathbb{N}. A_i$ is finite, then $\bigcap_{i=0}^{\infty} A_i$ is finite;
6. p_6 : if $\forall i \in \mathbb{N}. A_i$ is infinite, then $\bigcap_{i=0}^{\infty} A_i$ is finite;
7. p_7 : if $\forall i \in \mathbb{N}. A_i$ is infinite, then $\bigcap_{i=0}^{\infty} A_i$ is infinite.

3.1 Answer

Write your answer here.