Computability Assignment
Year 2013/14 - Number 10

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1 Preliminaries

Recall that for $a, b \in \mathbb{N}$, $min\{a, b\}$ is the least element between a and b. Recall also that a set $C \subseteq \mathbb{N}$ is called *upward closed* iff $\forall x \in C.\ \forall y \in \mathbb{N}\ (y > x \implies y \in C)$.

2 Question

Let $g,\ h \in \mathcal{R}$, and define

$$f(x) = \begin{cases} g(x) & \textit{whenever } x \in \mathsf{K} \\ \min\{g(x), h(x)\} + 1 & \textit{otherwise} \end{cases}$$

Is it possible to find g and h such that $f \in \mathcal{R}$ and total? If it is so, provide g, h, and the proof that $f \in \mathcal{R}$ and total; otherwise, provide a proof of why $f \notin \mathcal{R}$ or not total regardless of the choice of g and h.

2.1 Answer

Write your answer here.

3 Question

Prove or disprove: there exists an upward closed set $C \notin \mathcal{RE}$.

3.1 Answer

Write your answer here.

4 Question

Prove or disprove: the function f defined below belongs to $\mathcal{R}$.

$$f(n) = \begin{cases} (\varphi_n(n))^n & whenever\ \varphi_n(n)\ is\ defined \\ 77 & otherwise \end{cases}$$