Computability Assignment
Year 2012/13 - Number 10

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1 Question

Prove that the set

$$A = \{n \mid \phi_n(5+n) = 7\}$$

is $\mathcal{RE}$.

1.1 Answer

The set is semi-recursive, indeed the semi-characteristic function for A is

$\tilde{\chi}_A = \begin{cases} 1 & \phi_n(5+n) = 7 \\ undefined & ow \end{cases}$

Sum and comparison (with a potentially undefined operand) are semirecursive, and $\phi_n(m)$ is also semirecursive.

(RZ: not really a proof, it relies on A being RE)

2 Question

Prove that the set A defined above is **not** recursive, following the sketch below:

1. Prove that $g(n, x) = 7 \cdot \tilde{\chi}_{\mathsf{K}}(n)$ is a recursive partial function.
2. Prove that $f(n) = \# \left(\lambda x. \begin{cases} 7 & \text{if } n \in \mathsf{K} \\ undefined & \text{otherwise} \end{cases} \right)$ is a recursive total function.

3. Prove that $\chi_{\mathsf{K}}(n) = \chi_A(f(n))$ for all n. (If $n \in \mathsf{K}$ then ... If $n \notin \mathsf{K}$ then ...)
4. Prove that if A were recursive, then the set K would be recursive as well.
5. Conclude that A can not be recursive.

2.1 Answer

1. $\tilde{\chi}_{\mathsf{K}}(n)$ is a recursive partial function. The additional multiplication poses no problems to the recursivity of the new function
2. f is recursive, indeed it can be implemented in this way:

```
Func<int, int> f(int n)
{
        return x =>
        {
        var dummy = eval1(n, x); // If this is invocation is undefined, this
        return 7;
        };
}
```

   The returned function can be successfully assembled because of the s-m-n theorem.

3. If $n \in \mathsf{K}$ then the program produced by f will return 7, and the characteristic function will return 1. Otherwise, it will loop forever ($undefined$), and the characteristic function will return 0.
4. Since we proved that, given n, the two characteristic functions do exist and behave in the same way, a verifier for A would imply that a verifier for K would also exist.
5. We already know that K is not recursive, so the assumption that A is recursive is incorrect.

3 Question

Prove whether the set $\bar{A}$ is $\mathcal{RE}$, with A as defined above.

3.1 Answer

If a semiverifier for $\bar{A}$ existed, it would be possible to create a complete verifier for A (using the semiverifier for A), but we already know that A is not recursive.