Computability Assignment
Year 2012/13 - Number 9

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1 Question

Assume that f is a recursive partial function satisfying the following property:

$$\forall n, m \in \mathbb{N}.\ f(2^n 3^m) = \begin{cases} m+1 & \text{if } \phi_n(m) = 0 \\ 0 & \text{otherwise} \end{cases}$$

(Note that the above makes no guarantees on e.g. what $f(7)$ actually is).
Prove that:

1. The set $A = \{n \mid \phi_n(5 \cdot n) = 0\}$ is recursive.
2. (Harder, feel free to skip it)
   The set $B = \{n \mid \phi_n(2) = 5\}$ is recursive.

(Note: actually f is a non recursive function, and A, B are non recursive sets. Still, I'm interested in how one proves the above portion of a reduction argument.)

1.1 Answer

$$\chi_A(n) = \begin{cases} 1 & \text{if } f(2^n 3^{5n}) = 0, \\ 0 & \text{otherwise.} \end{cases}$$

From the characteristic function $\chi_A(n)$, it is easy to see that the set A is recursive, since $1 \in \mathcal{R}$, $0 \in \mathcal{R}$ and $f(2^n 3^{5n}) = 0$ is recursive as well.

(RZ: ok, note that $f(2^n 3^{5n}) = 0$ is recursive because f is recursive and $f(2^n 3^m)$ is defined. If the latter could be undefined, then that predicate would not be recursive.)