Computability Assignment
Year 2012/13 - Number 9

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1 Question

Assume that f is a recursive partial function satisfying the following property:

$$\forall n, m \in \mathbb{N}.\ f(2^n 3^m) = \begin{cases} m+1 & \text{if } \phi_n(m) = 0 \\ 0 & \text{otherwise} \end{cases}$$

(Note that the above makes no guarantees on e.g. what $f(7)$ actually is).

Prove that:

1. The set $A = \{n \mid \phi_n(5 \cdot n) = 0\}$ is recursive.
2. (Harder, feel free to skip it)
   The set $B = \{n \mid \phi_n(2) = 5\}$ is recursive.

(Note: actually f is a non recursive function, and A, B are non recursive sets. Still, I'm interested in how one proves the above portion of a reduction argument.)

1.1 Answer

If the set A is recursive , exists X_A(x) that belongs to R. (RZ: you need the opposite direction here) We consider the following implementation of X_A:

$X_A(x) = Cond(Eq(f(2^x 3^{5x}), 0), 0, 1)$

(RZ: OK, more details about why this is true?)

Since $X_A(x)$ is a composition of functions that belong to R, we can conclude $X_A$belongs to R. Hence, $X_A \in R \implies A \in R$.