Computability Assignment
Year 2012/13 - Number 9

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1 Question

Assume that f is a recursive partial function satisfying the following property:

$$\forall n, m \in \mathbb{N}.\ f(2^n 3^m) = \begin{cases} m+1 & \text{if } \phi_n(m) = 0 \\ 0 & \text{otherwise} \end{cases}$$

(Note that the above makes no guarantees on e.g. what $f(7)$ actually is).

Prove that:

1. The set $A = \{n \mid \phi_n(5 \cdot n) = 0\}$ is recursive.
2. (Harder, feel free to skip it)
   The set $B = \{n \mid \phi_n(2) = 5\}$ is recursive.

(Note: actually f is a non recursive function, and A, B are non recursive sets. Still, I'm interested in how one proves the above portion of a reduction argument.)

1.1 Answer

If we assume that f is recursive, A and B are recursive sets, because we can provide a verifier for them.

1. A possible verifier for A is$V_A(x) = \begin{cases} 1 & if\ f\left(2^x 3^{5x}\right) > 0 \\ 0 & otherwise \end{cases}$

   (RZ: don't confuse a characteristic function with a verifier (which is a program implementing the char.function))

2. A possible verifier for B is $V_B(x) = \begin{cases} 1 & if\ f\left(2^{\#\left(\lambda y.\begin{cases}0 & \phi_x(y)=5\\ 1 & ow\end{cases}\right)} 3^2\right) > 0 \\ 0 & otherwise \end{cases}$

(RZ: nice idea but $\lambda y.\begin{cases}0 & \phi_x(y)=5\\ 1 & ow\end{cases}$ is not recursive, so you can't use that inside the "#" !)