Computability Assignment
Year 2012/13 - Number 8

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1 Question

Prove that the following set is **not** λ-definable.

$$A = \{\#M \mid \exists n \in \mathbb{N}.\ M\ulcorner n\urcorner =_{\beta\eta} \ulcorner 5\urcorner\}$$

1.1 Answer

Let's Apply Rice:
1)$\#M \in A and M =_{\beta\eta} N$ implies that $\exists n \in \mathbb{N}. M\ulcorner n\urcorner =_{\beta\eta} \ulcorner 5\urcorner and N\ulcorner n\urcorner =_{\beta\eta} \ulcorner 5\urcorner then\ \#N \in A$ so A is semantically closed
2) $\#(K\ \ulcorner 5\urcorner) \in A$
3) $\#\,\Omega\ \notin A$
For these reasons A is not λ- defineable

2 Question

Prove that the following set is semantically closed. Then, prove that it **is** λ-definable.

$$A = \{\#M \mid \forall N \in \Lambda.\ N\,M =_{\beta\eta} \mathbf{I}\}$$

2.1 Answer

$\#M \in A and\, M =_{\beta\eta} N' implies that \forall N \in \Lambda.\ NM\ =_{\beta\eta} I\ \implies\ NN' =_{\beta\eta} I$, so A is semantically closed
(RZ: so..?)

Note.

The following exercise is harder. Feel free to skip it.

3 Question

Prove **whether** the following set is λ-definable.

$$A = \{\#M \mid M\ulcorner M\urcorner =_{\beta\eta} M\}$$

(Note: there is at least one simple solution to this. You do not need to try huge formulae for this.)

3.1 Answer

Write your answer here.