Computability Assignment
Year 2012/13 - Number 8

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1 Question

Prove that the following set is **not** λ-definable.

$$A = \{\#M \mid \exists n \in \mathbb{N}.\ M\ulcorner n\urcorner =_{\beta\eta} \ulcorner 5\urcorner\}$$

1.1 Answer

To prove that we can use the Rice theorem.

1. The set A is semantically closed because. Take $\#M \in A$ and $M =_{\beta\eta} N$ for all programs M, N. So $\exists n\ M\ulcorner n\urcorner =_{\beta\eta} \ulcorner 5\urcorner$ but this implies that also $\exists n\ N\ulcorner n\urcorner =_{\beta\eta} \ulcorner 5\urcorner$ and so $\#N \in A$.
2. Second we need to prove that A is not empty. Just take $\#K\ulcorner 5\urcorner$: it belongs to the set because $K\ulcorner 5\urcorner\ulcorner m\urcorner =_{\beta\eta} \ulcorner 5\urcorner$ for any m (since it project out the first element).
3. Third we need to prove that $A \neq \mathbb{N}$. we can take $\#\Omega$. . Clearly $\#\Omega$ is not in the set A so the property is verified.

So the set is not λ-definable.

2 Question

Prove that the following set is semantically closed. Then, prove that it **is** λ-definable.

$$A = \{\#M \mid \forall N \in \Lambda.\ N\,M =_{\beta\eta} \mathbf{I}\}$$

2.1 Answer

Note.

The following exercise is harder. Feel free to skip it.

3 Question

Prove **whether** the following set is λ-definable.

$$A = \{\#M \mid M^{\ulcorner}M^{\urcorner} =_{\beta\eta} M\}$$

(Note: there is at least one simple solution to this. You do not need to try huge formulae for this.)

3.1 Answer

Write your answer here.