Computability Assignment
Year 2012/13 - Number 7

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1 Question

Prove that the following set is not λ-definable.

$$A = \{\#M \mid M \text{ has a } \beta\text{-normal form}\}$$

(Hint: show that, if A were λ-definable, then also K_λ would be λ-definable, hence obtaining a contradiction.)

1.1 Answer

$\mathsf{K}_\lambda = \{\#M \mid M \ulcorner M \urcorner \text{ has a } \beta\text{-normal form}\}$

I suppose it has something to do with the similarity between M and $M\ulcorner M \urcorner$, maybe I should ignore an argument, or provide the missing argument to translate one expression into the other, but I can't see the whole picture. I'm sorry.

2 Question

Let A be a λ-definable set. Prove that

$$B = (A \cup \{b_1, \ldots, b_n\}) \setminus \{c_1, \cdots, c_m\}$$

is also λ-definable.

(Hint: do not reinvent the results we saw in class, just apply them.)

2.1 Answer

If A is λ-definable, then there exists a verifier for$V_A$for it.

Then we can create a verifier for B in this way:

$V_B = \lambda x.And\,(Or\,(V_A\,x)\,(V_b\,x))\,(Not\,(V_c\,x))$

where $V_b$and V_c are verifiers for $\{b_1, \ldots, b_n\}$ and $\{c_1, \cdots, c_m\}$, respectively. They do exist, because these two sets are finite.

3 Question

Let A be a **non** λ-definable set. Prove that

$$B = (A \cup \{b_1, \ldots, b_n\}) \setminus \{c_1, \cdots, c_m\}$$

is also **non** λ-definable.

(Hint: prove the contrapositive. That is, prove that if B were λ-definable, then also A would be such.)

3.1 Answer

λ-definability is closed with respect to union, complement and intersection.$\{b_1, \ldots, b_n\}$ and $\{c_1, \cdots, c_m\}$ are λ-definable, so if B was λ-definable, A would necessarily have to be λ-definable, which is a contraddiction.

(RZ: correct intuition, but I am not convinced you understood how to justify the last step. In an exam, I would assign only partial points for this.)