Computability Assignment
Year 2012/13 - Number 7

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More information about assignments at [1]

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1 Question

Prove that the following set is not λ-definable.

$$A = \{\#M \mid M \text{ has a } \beta\text{-normal form}\}$$

(Hint: show that, if A were λ-definable, then also K_λ would be λ-definable, hence obtaining a contradiction.)

1.1 Answer

By contradiction. Say A is λ-defined by V_A. We can build a verifier for K_λ as follows,

$$V_{\mathsf{K}_\lambda} = \lambda n.V_A\ (\mathbf{App}\ n\ (\mathbf{Num}\ n))$$

(RZ: ok, in an exam I'll want you to state why V_{K_λ} works as expected)

2 Question

Let A be a λ-definable set. Prove that

$$B = (A \cup \{b_1, \dots, b_n\}) \setminus \{c_1, \cdots, c_m\}$$

is also λ-definable.

(Hint: do not reinvent the results we saw in class, just apply them.)

[1] http://disi.unitn.it/∼zunino/teaching/computability/assignments

2.1 Answer

Say A is λ-defined by V_A. We know the finite sets $B_0 = \{b_1, \cdots, b_n\}$, $C = \{c_1, \cdots c_m\}$ can be respectively defined by V_{B_0} and V_C. We can write a verifier for B since $B = (A \cup B_0) \backslash C$.

3 Question

Let A be a **non** λ-definable set. Prove that

$$B = (A \cup \{b_1, \ldots, b_n\}) \setminus \{c_1, \cdots, c_m\}$$

is also **non** λ-definable.

(Hint: prove the contrapositive. That is, prove that if B were λ-definable, then also A would be such.)

3.1 Answer

Let's try the contraposition. If B is λ-defined by V_B, then $B \backslash B_0$ is λ-definable. $A \cap B_0 \subseteq B_0$ and $A \cap C \subseteq C$ are finite sets, they are also definable. hence$A = (B \backslash B_0) \cup (A \cap B_0) \cup (A \cap C)$ is λ-definable.

(RZ: very good. During an exam it would be nice to have a short proof of the last set equality – a Venn diagram is enough)