Computability Assignment
Year 2012/13 - Number 6

Please keep this file anonymous: do not write your name inside this file.
More information about assignments at http://disi.unitn.it/∼zunino/teaching/computability/assignments
Please do not submit a file containing only the answers; edit this file, instead, filling the answer sections.

1 Question

Write a λ-term M implementing the following specification:

$$M \ulcorner n \urcorner = \ulcorner \lambda x_0 \dots x_n.\ x_n x_{n-1} \cdots x_0 \urcorner$$

(Note: The notation $\ulcorner n \urcorner$ above stands for the numeral n, while $\ulcorner N \urcorner$ stands for $\ulcorner \# N \urcorner$ – inside LyX it's hard to tell them apart, but will appear correctly in the PDFs)

1.1 Answer

```
let makeBody current max =
        match current with
                | max  -> Var current
                | _    -> App (makeBody (current + 1) max) (Var current)

let makeLambdas current max =
        match current with
                | max  -> Lam current (makeBody 0 max)
                | _    -> Lam current (makeLambdas (current + 1) max)

let m n = makeLambdas 0 n
```

$MakeBody = \Theta\lambda gcm.(Eq\, c\, m)\ (Var\ c)\ (App\ (g\ (Succ\, c)\, m)\ c)$
(RZ: the last c should be $Var\ c$, I think)
$MakeLambdas = \Theta\lambda gcm.(Eqcm)(Lamc(MakeBody\ulcorner 0\urcorner m))(Lamc(g(Succ 1)m))$

(RZ: maybe $Succ\ c$, not $Succ\ 1$)
$M = \lambda n.MakeLambdas\ \ulcorner 0 \urcorner\ n$

2 Question

Write a λ-term M which, when given as input $\ulcorner N \urcorner$, evaluates to $\ulcorner O \urcorner$, where O is obtained from N by replacing every syntactic occurrence of Ω with $\mathbf{I}$.

To the purpose of this exercise, assume $\Omega = (\lambda x_0.x_0x_0)(\lambda x_0.x_0x_0)$ and $\mathbf{I} = \lambda x_0.x_0$.

For example, here are some expected outputs:

$$M\ \ulcorner \lambda x_5.\Omega \urcorner =_{\beta\eta} \ulcorner \lambda x_5.\mathbf{I} \urcorner$$
$$M\ \ulcorner \lambda x_3.\mathbf{K}\Omega \urcorner =_{\beta\eta} \ulcorner \lambda x_3.\mathbf{KI} \urcorner$$
$$M\ \ulcorner \lambda x_1.x_1\Omega(\lambda x_7.x_1\Omega) \urcorner =_{\beta\eta} \ulcorner \lambda x_1.x_1\mathbf{I}(\lambda x_7.x_1\mathbf{I}) \urcorner$$
$$M\ \ulcorner (\lambda x_0.x_0x_0)(\lambda x_0.x_0x_0) \urcorner =_{\beta\eta} \ulcorner \mathbf{I} \urcorner$$
$$M\ \ulcorner (\lambda x_1.x_1x_1)(\lambda x_1.x_1x_1) \urcorner =_{\beta\eta} \ulcorner (\lambda x_1.x_1x_1)(\lambda x_1.x_1x_1) \urcorner$$

Hint: use **Sd**, etc. as approprate.

2.1 Answer

M is a function that visits the whole syntax tree and returns an updated syntax tree. Applications that matchΩ are replaced with$\mathbf{I}$.

$V = \lambda gi.Var\ i$
$A = \lambda gmn.(Eq\ (App\ m\ n)\ \ulcorner \Omega \urcorner)\ \ulcorner \mathbf{I} \urcorner\ (App\ (g\ m)\ (g\ n))$
$L = \lambda gim.Lam\ i\ (g\ m)$
$M = \Theta\lambda gn.Sd\ n\ (V\ g)\ (A\ g)\ (L\ g)$

(RZ: ok, you could also have checked for Ω before using Sd. Looks correct anyway)