Computability Assignment
Year 2012/13 - Number 6

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Please do not submit a file containing only the answers; edit this file, instead, filling the answer sections.

1 Question

Write a λ-term M implementing the following specification:

$$M\ \ulcorner n \urcorner = \ulcorner \lambda x_0 \dots x_n.\ x_n x_{n-1} \cdots x_0 \urcorner$$

(Note: The notation $\ulcorner n \urcorner$ above stands for the numeral n, while $\ulcorner N \urcorner$ stands for $\ulcorner \#N \urcorner$ – inside LyX it's hard to tell them apart, but will appear correctly in the PDFs)

$Lam\ulcorner 0\urcorner(Lam\ulcorner n\urcorner\_Fst\_nG(Cons(Var\ulcorner n\urcorner)\_n))$

$G = \lambda p, Cons(App\_Fst(p)\_Var\ulcorner Pred(Sndp)\urcorner)\_(Predn)$

(RZ: this looks rather wrong. What about the other lambdas? Also, be careful because you don't want to use $\ulcorner - \urcorner$ here. Be more precise about where you put parenteses, many seem to be missing and/or put in the wrong places.)

1.1 Answer

Write your answer here.

2 Question

Write a λ-term M which, when given as input $\ulcorner N \urcorner$, evaluates to $\ulcorner O \urcorner$, where O is obtained from N by replacing every syntactic occurrence of Ω with $\mathbf{I}$.

To the purpose of this exercise, assume $\Omega = (\lambda x_0.x_0x_0)(\lambda x_0.x_0x_0)$ and $\mathbf{I} = \lambda x_0.x_0$.

For example, here are some expected outputs:

$M \ulcorner \lambda x_5.\Omega \urcorner =_{\beta\eta} \ulcorner \lambda x_5.\mathbf{I} \urcorner$
$M \ulcorner \lambda x_3.\mathbf{K}\Omega \urcorner =_{\beta\eta} \ulcorner \lambda x_3.\mathbf{KI} \urcorner$
$M \ulcorner \lambda x_1.x_1\Omega(\lambda x_7.x_1\Omega) \urcorner =_{\beta\eta} \ulcorner \lambda x_1.x_1\mathbf{I}(\lambda x_7.x_1\mathbf{I}) \urcorner$
$M \ulcorner (\lambda x_0.x_0x_0)(\lambda x_0.x_0x_0) \urcorner =_{\beta\eta} \ulcorner \mathbf{I} \urcorner$
$M \ulcorner (\lambda x_1.x_1x_1)(\lambda x_1.x_1x_1) \urcorner =_{\beta\eta} \ulcorner (\lambda x_1.x_1x_1)(\lambda x_1.x_1x_1) \urcorner$

Hint: use **Sd**, etc. as approprate.

2.1 Answer

$\ulcorner \#\Omega \urcorner = 449$
$M = \ominus(\lambda g.x.Eq\_x\_\ulcorner 449 \urcorner \ulcorner I \urcorner (Sd\_x\_V\_A\_L))$
$V = \lambda i.Var\_i$
$A = \lambda p.q.App\_(gp)\_(gq)$
$L = \lambda i.n.Lam\_i\_(gn)$