Computability Assignment
Year 2012/13 - Number 6

Please keep this file anonymous: do not write your name inside this file.
More information about assignments on the website[1].
Please do not submit a file containing only the answers; edit this file, instead, filling the answer sections.

1 Question

Write a λ-term M implementing the following specification:

$$M \ulcorner n \urcorner = \ulcorner \lambda x_0 \dots x_n.\ x_n x_{n-1} \cdots x_0 \urcorner$$

(Note: The notation $\ulcorner n \urcorner$ above stands for the numeral n, while $\ulcorner N \urcorner$ stands for $\ulcorner \# N \urcorner$ – inside LyX it's hard to tell them apart, but they will appear correctly in the PDFs)

1.1 Answer

F recursively creates $\ulcorner x_n x_{n-1} \cdots x_0 \urcorner$, G recursively adds the lambda abstraction part.

$$\begin{aligned} M &= \lambda n.G \ulcorner 0 \urcorner \\ G &= \mathbf{\Theta}\Big(\lambda g j.\mathbf{IsZero}\ (\mathbf{Sub}\ n\ j)\ (\mathbf{Lam}\ n\ (F \ulcorner 0 \urcorner))\ \Big(\mathbf{Lam}\ j\ (g\ (\mathbf{Succ}\ j))\Big)\Big) \\ F &= \mathbf{\Theta}\Big(\lambda f i.\mathbf{IsZero}\ (\mathbf{Sub}\ n\ i)\ (\mathbf{Var}\ n)\ \Big(\mathbf{App}\ (f\ (\mathbf{Succ}\ i))\ (\mathbf{Var}\ i)\Big)\Big) \end{aligned}$$

2 Question

Write a λ-term M which, when given as input $\ulcorner N \urcorner$, evaluates to $\ulcorner O \urcorner$, where O is obtained from N by replacing every syntactic occurrence of Ω with $\mathbf{I}$.

[1] http://disi.unitn.it/~zunino/teaching/computability/assignments

To the purpose of this exercise, assume $\Omega = (\lambda x_0.x_0x_0)(\lambda x_0.x_0x_0)$ and $\mathbf{I} = \lambda x_0.x_0$.

For example, here are some expected outputs:

$$M \ulcorner \lambda x_5.\Omega \urcorner =_{\beta\eta} \ulcorner \lambda x_5.\mathbf{I} \urcorner$$
$$M \ulcorner \lambda x_3.\mathbf{K}\Omega \urcorner =_{\beta\eta} \ulcorner \lambda x_3.\mathbf{KI} \urcorner$$
$$M \ulcorner \lambda x_1.x_1\Omega(\lambda x_7.x_1\Omega) \urcorner =_{\beta\eta} \ulcorner \lambda x_1.x_1\mathbf{I}(\lambda x_7.x_1\mathbf{I}) \urcorner$$
$$M \ulcorner (\lambda x_0.x_0x_0)(\lambda x_0.x_0x_0) \urcorner =_{\beta\eta} \ulcorner \mathbf{I} \urcorner$$
$$M \ulcorner (\lambda x_1.x_1x_1)(\lambda x_1.x_1x_1) \urcorner =_{\beta\eta} \ulcorner (\lambda x_1.x_1x_1)(\lambda x_1.x_1x_1) \urcorner$$

Hint: use **Sd**, etc. as appropriate.

2.1 Answer

We identify $\Omega = (\lambda x_0.x_0x_0)(\lambda x_0.x_0x_0)$ by its numeral code:$\#\Omega = 449$.

$$\begin{aligned} M &= \mathbf{\Theta}(\lambda en.\ \mathbf{Eq}\ n\ \ulcorner 449 \urcorner\ \ulcorner \mathbf{I} \urcorner\ (\mathbf{Sd}\ n\ V\ A\ L)) \\ V &= \lambda i.\mathbf{Var}\ i \\ A &= \lambda pq.\mathbf{App}\ e\ p\ (e\ q) \\ L &= \lambda im.\mathbf{Lam}\ i\ (e\ m) \end{aligned}$$

(RZ: should be $A = \lambda pq.\mathbf{App}\ (e\ p)\ (e\ q)$ above)