Computability Assignment
Year 2012/13 - Number 5

Please keep this file anonymous: do not write your name inside this file.

More information about assignments at http://disi.unitn.it/∼zunino/teaching/computability/assignments

Please do not submit a file containing only the answers; edit this file, instead, filling the answer sections.

1 Question

(I am re-proposing this exercise since only a few students solved it. This exercise is rather important, since it involves a reasoning which frequently appeared in past exam questions. While we shall see more examples of these concepts in class, it would be useful to start exercising on that. If you have already sumbitted an answer, skip this and do *not* resubmit your answer please.)

Let $\mathcal{F}$ be the set of partial functions $\{f \in (\mathbb{N} \rightsquigarrow \mathbb{N}) | \forall x \in \mathbb{N}.\ f(2 \cdot x) = x\}$.

- Define two distinct partial functions f_1, f_2 which belong to $\mathcal{F}$. (I.e, provide two such examples.)
- Define two distinct partial functions g_1, g_2 which do *not* belong to $\mathcal{F}$. (I.e, provide two such examples.)
- Define a partial function $f \in \mathcal{F}$, and consider the set of its *finite* restrictions $\mathcal{G} = \{g \in (\mathbb{N} \rightsquigarrow \mathbb{N}) | g \subseteq f \ \wedge\ \mathsf{dom}(g) \text{ finite}\}$.
  - Define two distinct partial functions h_1, h_2 which belong to $\mathcal{G}$. (I.e, provide two such examples.)
  - Prove whether $\mathcal{F} \cap \mathcal{G} = \emptyset$.

1.1 Answer

Write your answer here.

- $f_1(x) = \begin{cases} x/2 & x \text{ is even} \\ \mathsf{undef} & \text{otherwise} \end{cases}$, $f_2(x) = \begin{cases} x/2 & x \text{ is even} \\ x & x \text{ is odd} \end{cases}$; $\forall x \in \mathbb{N}. \begin{Bmatrix} f_1(2x) = x \Rightarrow f_1 \in \mathcal{F} \\ f_2(2x) = x \Rightarrow f_2 \in \mathcal{F} \end{Bmatrix}$

- $g_1(x) = x$, $g_2(x) = \mathsf{undef}$; $g_1(2) = 2 \neq 1 \Rightarrow g_1 \notin \mathcal{F}$; $g_2(2) = \mathsf{undef} \neq 1 \Rightarrow g_2 \notin \mathcal{F}$
- $f(x) = \begin{cases} x/2 & x \text{ is even} \\ \mathsf{undef} & \text{otherwise} \end{cases}$, $h_1(x) = \begin{cases} 1 & x = 2 \\ \mathsf{undef} & \text{otherwise} \end{cases}$, $h_2(x) = \begin{cases} 2 & x = 4 \\ \mathsf{undef} & \text{otherwise} \end{cases}$
  - $h_1(2) = 1 = f(2) \Rightarrow h_1 \subseteq f$; $h_2(4) = 2 = f(4) \Rightarrow h_2 \subseteq f$
  - $\mathcal{F} \cap \mathcal{G} = \emptyset$, since $\mathcal{F}$ contains only function defined (at least) on all even naturals;$\mathcal{G}$ instead contains only function with finite domain; there is no finite domain containing every and each even natural number

2 Question

Consider the following function:

$$f(n) = \sum_{i=0}^{n} i^2 + i$$

Write a FOR loop implementing f, then translate it in the λ-calculus as program F_1.

Then, write a recursive Java-like function implementing f, and translate it in the λ-calculus as program F_2.

2.1 Answer

Write your answer here.

1. $total := 0$
2. for $i \in [0..n]$do
   (a) $total += i^2 + i$

$F = \lambda n.n\,(\lambda it.\mathsf{Add}\,t\,(\mathsf{Add}\,i\,(\mathsf{Mul}\,i\,i)))\ulcorner 0 \urcorner \ulcorner 0 \urcorner$

(RZ: the idea is in part correct, but you need to pair 0,0 with a Cons or it does not work since nGb repeats G n times on *one* b, and not two. Also you need to loop $n + 1$ times.)

1. for $i \in [0..n]$ do
   (a) if $i > 0$
      i. then $i^2 + 1 + F(i - 1)$
      ii. else $total$

$F = \Theta\,(\lambda f.\,(\lambda it.\mathsf{IsZero}\,i\,t\,(Add\ \ (f\,(\mathsf{Pred}\,i)\,(\mathsf{Add}\,i\,(\mathsf{Mul}\,i\,i)))))\ulcorner 0 \urcorner \ulcorner 0 \urcorner)$

(RZ: no, this always passes $i = t = 0$ which is not what you want)

3 Question

Consider the following function:

$$f(n) = \begin{cases} x^2 + y & \text{if } n = \mathsf{pair}(\mathsf{inL}(x), y) \\ x + 4 \cdot y & \text{if } n = \mathsf{pair}(\mathsf{inR}(x), y) \end{cases}$$

Convince yourself that f is defined for all naturals n, i.e. it is total.

Write a λ-term implementing function f, exploiting the programs $Pair, Proj1, Proj2, InL, InR, Case, \ldots$ we saw in class (also defined in the notes).

3.1 Answer

Write your answer here.