

# Computability Assignment

## Year 2012/13 - Number 5

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**Please do not submit a file containing only the answers; edit this file, instead, filling the answer sections.**

### 1 Question

(I am re-proposing this exercise since only a few students solved it. This exercise is rather important, since it involves a reasoning which frequently appeared in past exam questions. While we shall see more examples of these concepts in class, it would be useful to start exercising on that. If you have already submitted an answer, skip this and do *not* resubmit your answer please.)

Let  $\mathcal{F}$  be the set of partial functions  $\{f \in (\mathbb{N} \rightsquigarrow \mathbb{N}) \mid \forall x \in \mathbb{N}. f(2 \cdot x) = x\}$ .

- Define two distinct partial functions  $f_1, f_2$  which belong to  $\mathcal{F}$ . (I.e, provide two such examples.)
- Define two distinct partial functions  $g_1, g_2$  which do *not* belong to  $\mathcal{F}$ . (I.e, provide two such examples.)
- Define a partial function  $f \in \mathcal{F}$ , and consider the set of its *finite* restrictions  $\mathcal{G} = \{g \in (\mathbb{N} \rightsquigarrow \mathbb{N}) \mid g \subseteq f \wedge \text{dom}(g) \text{ finite}\}$ .
  - Define two distinct partial functions  $h_1, h_2$  which belong to  $\mathcal{G}$ . (I.e, provide two such examples.)
  - Prove whether  $\mathcal{F} \cap \mathcal{G} = \emptyset$ .

#### 1.1 Answer

Write your answer here.

## 2 Question

Consider the following function:

$$f(n) = \sum_{i=0}^n i^2 + i$$

Write a FOR loop implementing  $f$ , then translate it in the  $\lambda$ -calculus as program  $F_1$ .

Then, write a recursive Java-like function implementing  $f$ , and translate it in the  $\lambda$ -calculus as program  $F_2$ .

### 2.1 Answer

For loop:

```
x:=0
int f(int n){
  for(i=0;i ≤ n;i++)
    {x=x+i*i+i;
    }
  return x;
}
```

**$\lambda$ -calculus:**  $G = \lambda p, Con\_ (Add\_ (Fstp)\_ (Add\_ (Mul\_ (Sndp)\_ (Sndp))\_ (Sndp)))\_ (Succ(Sndp))$

$F = \lambda n. Fst(nG(Con(\ulcorner 0 \urcorner \_ \ulcorner 1 \urcorner)))$

**Recursive function** int R (int n){  
 if (n:=0)  
 return 0;  
 else  
 return n\*n+n+R(n-1);  
}

$R = \odot(\lambda g. \lambda n, Iszero\_n\_ \ulcorner 0 \urcorner \_ Add\_n\_ (Add\_ (Mul\_n\_n)\_ g(Predn)))$

## 3 Question

Consider the following function:

$$f(n) = \begin{cases} x^2 + y & \text{if } n = \text{pair}(\text{inL}(x), y) \\ x + 4 \cdot y & \text{if } n = \text{pair}(\text{inR}(x), y) \end{cases}$$

Convince yourself that  $f$  is defined for all naturals  $n$ , i.e. it is total.

Write a  $\lambda$ -term implementing function  $f$ , exploiting the programs  $Pair, Proj1, Proj2, InL, InR, Case, \dots$  we saw in class (also defined in the notes).

### 3.1 Answer

$P = \lambda n, \text{case\_Proj1}(n)_{-}(\lambda i, \text{Add\_Proj2}(n)_{-}(\text{Mul\_i\_i})_{-}(\lambda m, \text{Add\_m\_}(\text{Mul\_}\ulcorner 4 \urcorner_{-}\text{Proj2}(n))))$   
 $\text{Proj1}(n) = \text{InL}(i)_{-}\text{or\_InR}(m)$   
(RZ: ok; watch out for parentheses, it's  $(\text{Proj2 } n)$  for instance)