Computability Assignment
Year 2012/13 - Number 3

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Please do not submit a file containing only the answers; edit this file, instead, filling the answer sections.

1 Question

Let A, B be sets, and let $\mathsf{id}_A, \mathsf{id}_B$ denote the identity functions over A and B respectively. Assume $f \in (A \to B)$ and $g \in (B \to A)$ be functions satisfying $g \circ f = \mathsf{id}_A$ and $f \circ g = \mathsf{id}_B$. Prove that f is a bijection (i.e., injective and surjective).

1.1 Answer

We know that $Dom(f) = A$ and $Range(g) = A$. This means that $f \circ g = f(g(x)) = \mathsf{id}_B \Longrightarrow f$ is surjective, because every element in set B has at least one corresponding element in set A. Using the same argumentation $g \circ f = g(f(x)) = \mathsf{id}_A \Longrightarrow g$ is surjective. Using the definition of inverse fuction we can write $g = f^{-1}$. $f^{-1}$is surjective , we conclude that f is bijection.

2 Question

Let A, B be sets, and let $f \in (A \leftrightarrow B)$ be a bijection. Define a bijection $g \in (\mathcal{P}(A) \leftrightarrow \mathcal{P}(B))$ and prove it is such.

2.1 Answer

Let $A' \subseteq A$ and $B' \subseteq B$. If $\forall a \in A \smallsetminus A'.\ f(a) \notin B'$, then $\exists g \in (A' \longleftrightarrow B')$. One example of such fuction g is fuction $f(A' \longleftrightarrow B')$. The condision set in the defition ensure us that the one-to-one correspondence is maintained between the subsets of A and B.

3 Question

Let A, B be two sets, and let $b \notin B$. Define a bijection f between the set of partial functions $(A \rightsquigarrow B)$ and the set of total functions $(A \to B \cup \{b\})$. Prove that is is such.

3.1 Answer

Write your answer here.

Note.

The exercises below are harder. Feel free to skip them if you find them too hard.

4 Question

Define a bijection $f \in [(\mathcal{P}(A) \times \mathcal{P}(B)) \leftrightarrow \mathcal{P}(A \uplus B)]$. Prove that is is such.

4.1 Answer

Write your answer here.

5 Question

Define a bijection $f \in [((A \uplus B) \to C) \leftrightarrow ((A \to C) \times (B \to C))]$. Prove that is is such.

5.1 Answer

Write your answer here.

6 Question

Define a bijection $f \in [((A \to (B \times C)) \leftrightarrow ((A \to B) \times (A \to C))]$. Prove that is is such.

6.1 Answer

Write your answer here.