Computability Assignment Year 2012/13 - Number 3

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1 Question

Let A, B be sets, and let id_A, id_B denote the identity functions over A and B respectively. Assume $f \in (A \to B)$ and $g \in (B \to A)$ be functions satisfying $g \circ f = id_A$ and $f \circ g = id_B$. Prove that f is a bijection (i.e., injective and surjective).

1.1 Answer

INJ

By contradiction we assume f not injective. So we will have at least two points in the dom(f) = A which are mapped into the same point in the ran(f) = B. This contradicts the fact that $g \circ f = id_A$ because we will have, i.e., g(f(x)) = x and also g(f(x')) = x where the latter does not return the value x', and so this is not the identity function. Furthermore we have that g is not a function.

SURJ

By contraddiction we assume f not surjective. So we will have at least one point in the ran(f) = B which is not reach by f. This contraddicts the fact that $f \circ g = id_B$ because g will be a partial function over its domain.

We can conclude saying that f is a bijection.

2 Question

Let A, B be sets, and let $f \in (A \leftrightarrow B)$ be a bijection. Define a bijection $g \in (\mathcal{P}(A) \leftrightarrow \mathcal{P}(B))$ and prove it is such.

2.1 Answer

Let $g \in (\mathcal{P}(A) \leftrightarrow \mathcal{P}(B))$ where $\mathcal{P}(B) = \{f(a) \mid a \in \mathcal{P}(A)\}$. Since f is a bijection then g is also a bijection.

3 Question

Let A, B be two sets, and let $b \notin B$. Define a bijection f between the set of partial functions $(A \rightsquigarrow B)$ and the set of total functions $(A \rightarrow B \cup \{b\})$. Prove that is is such.

3.1 Answer

Write your answer here.

Note.

The exercises below are harder. Feel free to skip them if you find them too hard.

4 Question

Define a bijection $f \in [(\mathcal{P}(A) \times \mathcal{P}(B)) \leftrightarrow \mathcal{P}(A \uplus B)]$. Prove that is is such.

4.1 Answer

Write your answer here.

5 Question

Define a bijection $f \in [((A \uplus B) \to C) \leftrightarrow ((A \to C) \times (B \to C))]$. Prove that is is such.

5.1 Answer

Write your answer here.

6 Question

Define a bijection $f \in [((A \to (B \times C)) \leftrightarrow ((A \to B) \times (A \to C))]$. Prove that is is such.

6.1 Answer

Write your answer here.