# Computability Assignment Year 2012/13 - Number 3 

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## 1 Question

Let $A, B$ be sets, and let $\mathrm{id}_{A}, \mathrm{id}_{B}$ denote the identity functions over $A$ and $B$ respectively. Assume $f \in(A \rightarrow B)$ and $g \in(B \rightarrow A)$ be functions satisfying $g \circ f=\operatorname{id}_{A}$ and $f \circ g=\operatorname{id}_{B}$. Prove that $f$ is a bijection (i.e., injective and surjective).

### 1.1 Answer

INJ
By contraddiction we assume $f$ not injective. So we will have at least two points in the $\operatorname{dom}(f)=A$ which are mapped into the same point in the $\operatorname{ran}(f)=$ $B$. This contraddicts the fact that $g \circ f=\operatorname{id}_{A}$ because we will have, i.e., $g(f(x))=x$ and also $g\left(f\left(x^{\prime}\right)\right)=x$ where the latter does not return the value $x^{\prime}$, and so this is not the identity function. Furthermore we have that $g$ is not a function.
SURJ
By contraddiction we assume $f$ not surjective. So we will have at least one point in the $\operatorname{ran}(f)=B$ which is not reach by $f$. This contraddicts the fact that $f \circ g=\operatorname{id}_{B}$ because $g$ will be a partial function over its domain.

We can conclude saying that $f$ is a bijection.

## 2 Question

Let $A, B$ be sets, and let $f \in(A \leftrightarrow B)$ be a bijection. Define a bijection $g \in(\mathcal{P}(A) \leftrightarrow \mathcal{P}(B))$ and prove it is such.

### 2.1 Answer

Let $g \in(\mathcal{P}(A) \leftrightarrow \mathcal{P}(B))$ where $\mathcal{P}(B)=\{f(a) \mid a \in \mathcal{P}(A)\}$. Since $f$ is a bijection then $g$ is also a bijection.

## 3 Question

Let $A, B$ be two sets, and let $b \notin B$. Define a bijection $f$ between the set of partial functions $(A \rightsquigarrow B)$ and the set of total functions $(A \rightarrow B \cup\{b\})$. Prove that is is such.

### 3.1 Answer

Write your answer here.

## Note.

The exercises below are harder. Feel free to skip them if you find them too hard.

## 4 Question

Define a bijection $f \in[(\mathcal{P}(A) \times \mathcal{P}(B)) \leftrightarrow \mathcal{P}(A \uplus B)]$. Prove that is is such.

### 4.1 Answer

Write your answer here.

## 5 Question

Define a bijection $f \in[((A \uplus B) \rightarrow C) \leftrightarrow((A \rightarrow C) \times(B \rightarrow C))]$. Prove that is is such.

### 5.1 Answer

Write your answer here.

## 6 Question

Define a bijection $f \in[((A \rightarrow(B \times C)) \leftrightarrow((A \rightarrow B) \times(A \rightarrow C))]$. Prove that is is such.

### 6.1 Answer

Write your answer here.

