Computability Assignment
Year 2012/13 - Number 3

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Please do not submit a file containing only the answers; edit this file, instead, filling the answer sections.

1 Question

Let A, B be sets, and let $\mathsf{id}_A, \mathsf{id}_B$ denote the identity functions over A and B respectively. Assume

$$f \in (A \to B)$$

and $g \in (B \to A)$ be functions satisfying $g \circ f = \mathsf{id}_A$ and $f \circ g = \mathsf{id}_B$. Prove that f is a bijection (i.e., injective and surjective).

1.1 Answer

1-Lets assume the function is injective:by contradiction

if it's not injective:not one to one ,$\exists(a, b).f(a) = f(b) \nRightarrow a = b$

$\exists a \neq b.f(a) = f(b)$

however , $f(a) = f(b)$ and $a \neq b$, then it is not possible $g(f(a)) = a$ and $g(f(b)) = b$,but $g \circ f = \mathsf{id}_A$

so: f is injective

2. Lets assume the function is surjective:by contradiction

if it's not surjective:onto, $\exists b \in B.\forall a \in A.f(a) \neq b$

but $f \circ g = f(g(b)) = \mathsf{id}_B = b$,

contradiction: f is surjective

since it's injective and surjective ,so it is bijective.

2 Question

Let A, B be sets, and let $f \in (A \leftrightarrow B)$ be a bijection. Define a bijection $g \in (\mathcal{P}(A) \leftrightarrow \mathcal{P}(B))$ and prove it is such.

2.1 Answer

assume that we have $A \in \mathbb{N}$, $B \in \mathbb{N}$

lets assume $f(x) = x + 1$

$g(x) = x + 2$

let's take$\mathcal{P}(A) = \{0, 1, 2\}, \mathcal{P}(B) = \{2, 3, 4\}$, and power set of A and B are subset of A,B

the $\mathcal{P}(A)$ to $\mathcal{P}(B)$are a bijection since any number in A will be mapped to one number in B

(RZ: ok, but what in the general case?)

3 Question

Let A, B be two sets, and let $b \notin B$. Define a bijection f between the set of partial functions $(A \rightsquigarrow B)$ and the set of total functions $(A \to B \cup \{b\})$. Prove that is is such.

3.1 Answer

Write your answer here.

Note.

The exercises below are harder. Feel free to skip them if you find them too hard.

4 Question

Define a bijection $f \in [(\mathcal{P}(A) \times \mathcal{P}(B)) \leftrightarrow \mathcal{P}(A \uplus B)]$. Prove that is is such.

4.1 Answer

Write your answer here.

5 Question

Define a bijection $f \in [((A \uplus B) \to C) \leftrightarrow ((A \to C) \times (B \to C))]$. Prove that is is such.

5.1 Answer

Write your answer here.

6 Question

Define a bijection $f \in [((A \to (B \times C)) \leftrightarrow ((A \to B) \times (A \to C))]$. Prove that is is such.

6.1 Answer

Write your answer here.