# Computability Assignment
# Year 2012/13 - Number 3

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## 1 Question

Let $A, B$ be sets, and let $\mathsf{id}_A, \mathsf{id}_B$ denote the identity functions over $A$ and $B$ respectively. Assume $f \in (A \to B)$ and $g \in (B \to A)$ be functions satisfying $g \circ f = \mathsf{id}_A$ and $f \circ g = \mathsf{id}_B$. Prove that $f$ is a bijection (i.e., injective and surjective).

### 1.1 Answer

injective:
assume $f(x) = f(y)$,$x, y \in A$
then $g(f(x)) = g(f(y))$
as $g \circ f = \mathsf{id}_A$, $id_A(x) = x$
$g(f(x)) = x = g(f(y)) = y$
So $x = y$ $f$ is injective.

**surjective:**$\forall z \in B \quad id_B(z) = f(g(z)) = z$
so $\exists a \in A$, take $a = g(z)$,$f(a) = z$
So $f$ is surjective.

## 2 Question

Let $A, B$ be sets, and let $f \in (A \leftrightarrow B)$ be a bijection. Define a bijection $g \in (\mathcal{P}(A) \leftrightarrow \mathcal{P}(B))$ and prove it is such.

### 2.1 Answer

$g = \{(A^{'}, B^{'})|A^{'} \subseteq A, \forall a \in A^{'}, B = \{b|b = f(a)\}\}$
(RZ: the $\forall$ is wrong, you actually want $g = \{(A^{'}, B^{'})|A^{'} \subseteq A, B = \{b \mid \exists a \in A'.\ b = f(a)\}\}$)

injective: assume $g(C) = g(D)$, $C, D \subseteq A$
$g(C) = \{b|\forall a \in C, b = f(a)\}$
$g(D) = \{b|\forall a \in D, b = f(a)\}$
as $f$ is bijection, all elements in $C, D$ are same
so $C = D$ $g$ is injective
surjective:for any $b \in B^{'} \subseteq B$
$a = f^{-}(b)$
$\exists A^{'} = \{a|a = f^{-}(b)\}$,$g(A^{'}) = B$
so $g$ is suijective

## 3 Question

Let $A, B$ be two sets, and let $b \notin B$. Define a bijection $f$ between the set of partial functions $(A \rightsquigarrow B)$ and the set of total functions $(A \to B \cup \{b\})$. Prove that is is such.

### 3.1 Answer

$f = \{(g, t)|g \subseteq A \times B, \exists A^{'} \subseteq A, g \in (A^{'} \to B).\forall a \in A, if a \in A^{'}, t = g, else g(a) = b\}$

(RZ: maybe something like
$f = \{(g, t)|g \subseteq A \times B, \exists A^{'} \subseteq A, g \in (A^{'} \to B).\forall a \in A, if\ a \in A^{'},\ t(a) = g(a), else\ t(a) = b\})$

for each $g$,there is only one $t$,which is equal to $g$,if $a \in A^{'}$
for each $t$,$\exists g$,which is equal to $t$ in the case of $a \in A^{'}$

# Note.

The exercises below are harder. Feel free to skip them if you find them too hard.

## 4 Question

Define a bijection $f \in [(\mathcal{P}(A) \times \mathcal{P}(B)) \leftrightarrow \mathcal{P}(A \uplus B)]$. Prove that is is such.

### 4.1 Answer

$f = \{((C, D), E)|C \subseteq A, D \subseteq B, E = \{(0, a)|a \in C\} \cup \{(1, b)|b \in D\}\}$

injective: for,each $(C, D)$,there is only one $E$ whose elements of $a, b$ composing $C, D$ respectively.

surjective:for each E, there is $(C, D)$which are made up of $a, b$ in E.

# 5 Question

Define a bijection $f \in [((A \uplus B) \to C) \leftrightarrow ((A \to C) \times (B \to C))]$. Prove that is is such.

## 5.1 Answer

$f = \{((< 0, a >< 1, b >), c), (< a, c >, < b, c >)|a \in A, b \in B, c \in C\}$

proof is the similar with above

(RZ: no)

# 6 Question

Define a bijection $f \in [((A \to (B \times C)) \leftrightarrow ((A \to B) \times (A \to C))]$. Prove that is is such.

## 6.1 Answer

$f = \{(a, (b, c)), ((a, b), (a, c))|a \in A, b \in B, c \in C\}$

proof is the similar with above

(RZ: no)