

# Computability Assignment

## Year 2012/13 - Number 1

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### 1 Question

Define a binary property  $p(x, y)$  over natural numbers such that we have both

1.  $\forall x \in \mathbb{N}. \exists y \in \mathbb{N}. p(x, y)$
2.  $\neg \exists y \in \mathbb{N}. \forall x \in \mathbb{N}. p(x, y)$

Provide a definition for  $p$ , and a proof for the above claims.

#### 1.1 Answer

$$p(x, y) = \begin{cases} 1 & x + 3 = y \\ 0 & \text{otherwise} \end{cases}$$

Given  $x$ , a value  $y$  that satisfies  $p$  exists. So statement 1 is true.

Now we try to demonstrate that an  $y$  that satisfies the property  $p$  for all the value of  $x$ . We call this value  $z$ . But not all the  $x$  have the same value of  $z$ , because if  $z = x$ , then  $p(z+2, z)$  is false. So claim 2 is true.