Computability Assignment
Year 2012/13 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers such that we have both

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$
2. $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

RZ: p is required to be a property over natural numbers

$$p(x, y) = \begin{cases} x = y, & y \in \mathbb{N} \\ x = \lfloor y \rfloor, & y \notin \mathbb{N} \end{cases}$$

Proof:

Let assume we have two sets of numbers such that $x \in \mathbb{N}$ and $y \in \mathbb{R}$, and we would like to find some binary property that maps x to y and viceversa by the binary equality operator, in such a way that the two condition holds.

- for all $x \in \mathbb{N}$, there exist a $y \in \mathbb{R}$ such that $x = y$, and 1 holds since $\mathbb{N} \subset \mathbb{R}$.
- assume $y \notin \mathbb{N}$ (because $y \in \mathbb{R}$), if we take $\lfloor y \rfloor$ (the floor function of y), we still have an equality with all values of x, therefore 2 holds. RZ: this does not prove 2