Computability Assignment
Year 2012/13 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers such that we have both

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$
2. $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

Let define p(x,y) as follow:

$$p(x, y) = \begin{cases} true & x > y \\ false & o.w. \end{cases}$$

$\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$ is respected. Assume to have$x = 0$, we can take $y = 1$. More generally speaking with $x = n$, taking $y = n + 1$ will always let us to leave the first formula respected.

For what regards the second formula, we can perform these steps:

$\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y) \iff \forall y \in \mathbb{N}.\neg\forall x \in \mathbb{N}.p(x, y) \iff \forall y \in \mathbb{N}.\exists x \in \mathbb{N}.\neg p(x, y)$

Then we have that the negation of our property is:

$$\neg p(x, y) = \begin{cases} true & x \leq y \\ false & o.w. \end{cases}$$

So if $y = 0$, x can be choosen as equal to y, which satisfies the negation of our property. Taking $x = y$ will always let us to leave the second formula respected.