Computability Assignment
Year 2012/13 - Number 1

Please keep this file anonymous: do not write your name inside this file.
More information about assignments at http://disi.unitn.it/∼zunino/teaching/computability/assignments

1 Question

Define a binary property $p(x, y)$ over natural numbers such that we have both

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$
2. $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

Define $p(x, y)$ as follow $p(x, y) = (x = y)$ in this case both 1. and 2. are satisfied.

1.1.1 $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$

RZ: ok, but you don't really need induction to prove $x = x$

The property is satisfied for $x = 0$ because exists an y that makes it true $y = 0$ in fact $p(0, 0) = (0 = 0)$ that it's true.

Now we assume that the theorem it's true for the generic x so we see that $y = x$ makes the preposition true.

The last step show us that for $x + 1$ the the preposition is true in fact is satisfied by $y = x + 1$:

if $y = x + 1$

$p(y, x + 1) = (y = x + 1)$ we sobstitute y by the rules above

$p(x + 1, x + 1) = (x + 1 = x + 1)$ so is satisfied.

Inductively we have proof the first claim.

1.1.2 $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$

If we choose $y = 0$ and $x = 0$ the property is verified but not for $y = 0$ and $x = 1$ so it isn't verified for all the x.

We can now take the generic y, the property is satisfied for x by the rules $x = y$ but not for $x + 1$ by the same rules in fact if $x = y$, $x + 1 \neq y$. For $y + 1$ the rules is the same as above in fact if $y = x$: $y + 1 = x + 1$ but $y + 1 \neq x + 2$ so $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$ is satisfied.