Computability Assignment
Year 2012/13 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers such that we have both

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$
2. $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

Let's take $p(x, y)$ such that $y = f(x) = 2x + 1$.

The first condition is respected by the fact that whichever x is chosen there will be always a y due to the fact that f is a total function.

The second condition can be proved by contradiction: Suppose that such y exists than must satisfy the property p for every x.

Let's pick $y = 1$ in order to satisfy the property $x = 0$ but to verify the for all try an other $x = 1$ then the property does not hold anymore.

But now is clear that if such ant y exists than the function f must not be injective, so to have all the x mapped to the same y while the chosen property at the beginning is clearly injective and makes impossible to have such y.