Computability Assignment
Year 2012/13 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers such that we have both

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$
2. $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

Write your answer here.

$p(x, y) = \begin{cases} true & y = 2x \\ false & otherwise \end{cases}$

1. is true because $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.y = 2x$
2. is true because $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.y = 2x$:
suppose $y = 2 \cdot (x)$ and $y = 2 \cdot (x + 1) \Rightarrow 2 \cdot (x) = 2 \cdot (x + 1) \Rightarrow x = x + 1 \Rightarrow$ $0 = 1 \Rightarrow$ ABSURD.