Computability Assignment
Year 2012/13 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers such that we have both

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$
2. $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

p(x,y) = x > y

Proof 1

Suppose $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.y > x \Longleftrightarrow \exists y \in \mathbb{N}.\forall x \in \mathbb{N}.y > x$

then exists a set Y = { y | take any x over $\mathbb{N} \Longrightarrow$ y > x} such that $\mathbb{N} \cup$ Y = $\mathbb{N}$ and $\mathbb{N} \cap$ Y = $\oslash$

RZ: what? I can't understand what Y should be

$\Longrightarrow$ Y = $\emptyset$

Proof 2

Suppose $\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.x > y$

Let x $\in \mathbb{N}$ and let y = x. RZ: you can not pick y here

In this case the property x>y is false