Computability Assignment
Year 2012/13 - Number 1

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1 Question

Define a binary property $p(x, y)$ over natural numbers such that we have both

1. $\forall x \in \mathbb{N}.\exists y \in \mathbb{N}.p(x, y)$
2. $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.p(x, y)$

Provide a definition for p, and a proof for the above claims.

1.1 Answer

Definition:$P(x, y) = \{(x, y) \mid x \in \mathbb{N} \wedge y \in \mathbb{N} \wedge y \geq x\}$

Proof:

For 1: $\forall x \in \mathbb{N}$,

let $y = x + 1$

so $\exists y \in \mathbb{N}, y \geq x$

For 2: $\forall y \in \mathbb{N}$,

let $x = y + 1$

so $\exists x \in \mathbb{N}, y \leq x$**.**

That is to say, $\neg\exists y \in \mathbb{N}.\forall x \in \mathbb{N}.P(x, y)$ **,**

for the reason $(\neg\exists x.p\,(x) \Leftrightarrow \forall x.\neg p\,(x))$**.**